I. EXPERIMENTAL DETAILS

While several samples demonstrated qualitatively similar behavior (non-Fermi-liquid exponents close to the critical point), we chose to present a consistent set of data measured on one sample with $r$ close enough to 1. We show two single-electron peaks, labeled peaks 1 and 2 in Fig. 3c of the main text, measured during two cooldowns. Specifically, Fig. 2 and 4c,d have been measured on Peak #2 at $\Delta V_{\text{gate}} = 0, 0.7, 1.2, 1.6, 2.0, 5.0 \text{ mV}$ (top to bottom).

Fig. S1 represents the conductance $G$ vs. bias voltage $V$ data corresponding to the vertical cross-sections of the maps in Fig. 2 of the main text. Both panels include the curve measured at the QPC (top curve) and several curves detuned from the QPC, either by the tunnel barrier asymmetry (panel A), or by shifting the level off-resonance with the gate voltage (panel B). Clearly, except for the special point of both symmetric coupling and on-resonance condition, the conductance is suppressed at small bias. (Note that the zero-bias suppression is limited at $eV \simeq k_B T$, leading to a saturation of the conductance at small $V$.) The similar trends shown in both panel demonstrate the equivalent role of the gate detuning and tunnel barrier asymmetry in destroying the quantum critical state.

In Fig. S2, we further analyze the data of Fig. 4d and present an empirical procedure to determine the width of the quantum critical region, alternative to the one discussed in the main text. We fit conductance $G(X)$ with the eq. (2) of the main text, replacing $X$ with an empirical expression $(X^2 + X_0^2)^{1/2}$. Thus introduced width, $X_0$, is the only fitting parameter for each curve. The extracted $X_0$ is plotted in the inset and is further fit by a power law dependence on $T$. The resulting exponent of approximately 0.7 is reasonably close to the expected $1/(1 + r)$.

II. DERIVATION OF THE HAMILTONIAN FOR A DISSIPATIVE SPINLESS QUANTUM DOT

We provide here the microscopic theory of our experiment, eventually leading us, in the following sections, to the inter-
acting Majorana resonant level Hamiltonian given in Eq. (1) of the main text.

Because of the large applied magnetic field, our experimental setup can be first modeled by a spinless quantum dot, described by a fermion operator $d^\dagger$, coupled to two conducting leads. The key feature of our device is the presence of an Ohmic dissipative environment, with sizeable quantum fluctuations of the voltage in the source and drain leads. The total Hamiltonian thus reads:

$$H = H_{\text{Dot}} + H_{\text{Leads}} + H_T + H_{\text{Env}},$$

where $H_{\text{Dot}} = e_d d^\dagger d$ is the Hamiltonian of the dot with the energy level $e_d$ (tuned by the backgate voltage $V_{\text{gate}}$), and $H_{\text{Leads}} = \sum_{\alpha=S,D} \sum_k \epsilon_k c^{\dagger}_{k\alpha} c_{k\alpha}$ represents the electrons in the source (S) and drain (D) leads. The most important piece of Hamiltonian (1) is $H_T$, which describes the tunneling between the dot and the leads with amplitudes $V_{S/D}$:

$$H_T = V_S \sum_k (c^{\dagger}_{kS} e^{-i\varphi_S} d + \text{h.c.}) + V_D \sum_k (c^{\dagger}_{kD} e^{i\varphi_D} d + \text{h.c.}),$$

where the operators $\varphi_{S/D}$ induce phase fluctuations of the tunneling amplitude between the dot and the S/D lead. These phase operators are canonically conjugate to the operators $Q_{S/D}$ corresponding to charge fluctuations on the S/D junctions. We follow the standard approach to treat quantum tunneling in the presence of a dissipative environment [1], which is valid for electrons propagating much slower than the electromagnetic field [2].

It is useful to transform to variables related to the total charge on the dot. To that end, we introduce [1] two new phase operators, $\varphi$ and $\psi$, related to the phases $\varphi_{S/D}$ by

$$\varphi_S = \kappa_S \varphi + \psi, \quad \varphi_D = \kappa_D \varphi - \psi,$$

where $\kappa_{S/D} = C_{S/D}/(C_S + C_D)$ in terms of the capacitances of the dots to the source/drain contacts, $C_{S/D}$. The phase $\varphi$ is the variable conjugate to the fluctuations of charge on the dot $Q_e = Q_S - Q_D$. Likewise, $\psi$ is the variable conjugate to $Q = (C_S Q_D + C_D Q_S)/(C_D + C_S)$. Assuming $C_S = C_D$, we have $\varphi_S = \varphi/2 + \psi$ and $\varphi_D = \varphi/2 - \psi$.

The gate voltage fluctuations can be disregarded in our experiment because the capacitance of the gate is negligible, $C_g \ll C_{S/D}$ (the opposite limit of a noisy gate coupled to a resonant level was considered in Refs. [3, 4]). In fact, the coupling of the fluctuations of the total charge on the dot to the environment is negligible [1]. Thus, only the relative phase difference between the two leads remains [1, 5], and the tunneling Hamiltonian becomes

$$H_T = V_S \sum_k (c^{\dagger}_{kS} e^{-i\frac{\varphi}{2}} d + \text{h.c.}) + V_D \sum_k (c^{\dagger}_{kD} e^{i\frac{\varphi}{2}} d + \text{h.c.}).$$

The last part of Eq. (1) is the Hamiltonian of the environment, $H_{\text{Env}}$ [1, 6, 7]. The environmental modes are represented by harmonic oscillators described by inductances and capacitances such that their frequencies are given by $\omega_k = 1/\sqrt{L_k C_k}$. These oscillators are then bilinearly coupled to the phase operator $\varphi$ through the oscillator phase:

$$H_{\text{Env}} = \frac{Q^2}{2C} + \sum_{k=1}^N \left[ \frac{q^2_{kS}}{2C_{kS}} + \left( \frac{h}{e} \right)^2 \frac{1}{2L_k} (\varphi - \varphi_k)^2 \right].$$

### III. Mapping onto the Luttinger Liquid Tunneling Problem

In this section, we first use bosonization [8] to map our model Eq. (1) to that of a resonant level contacted by two Luttinger liquids. In carrying out this mapping, we follow closely previous work on tunneling through a single barrier with an environment [9, 10] and the Kondo effect in the presence of resistive leads [5].

The two metallic leads in our case can be reduced to two semi-infinite one-dimensional free fermionic baths, which are non-chiral [8]. By unfolding them, one can obtain two chiral fields [8], which both couple to the dot at $x = 0$. We bosonize the fermionic fields in the standard way [8, 11] $c_{S/D}(x) = \frac{1}{\sqrt{2\pi a_0}} \exp[i\varphi_{S/D}(x)]$ (neglecting Klein factors for simplicity), where $\varphi_{S/D}$ are bosonic fields introduced to describe electronic states in the leads, and $a_0$ is a short time cutoff. Defining the flavor field $\phi_f$ and charge field $\phi_c$ by

$$\phi_f \equiv \frac{\varphi_S - \varphi_D}{\sqrt{2}}, \quad \phi_c \equiv \frac{\varphi_S + \varphi_D}{\sqrt{2}},$$

we can rewrite the Hamiltonian of the leads as

$$H_{\text{Leads}} = \frac{v_F}{4\pi} \int_{-\infty}^{+\infty} dx \left[ (\partial_x \phi_c)^2 + ([\partial_x \phi_f])^2 \right].$$

with $v_F$ the Fermi velocity. The tunneling Hamiltonian then becomes

$$H_T = \frac{V_S}{\sqrt{2\pi a_0}} \exp \left[ -i \phi_c(0) + \phi_f(0) \right] \frac{\sqrt{2}}{2} d + \text{h.c.} + \frac{V_D}{\sqrt{2\pi a_0}} \exp \left[ -i \phi_c(0) - \phi_f(0) \right] \frac{\sqrt{2}}{2} d + \text{h.c.}.$$ 

Note a key feature of $H_T$: the fields $\varphi$ and $\phi_f(0)$ enter in the same way in the tunnel process. Combining these two fields together embodies a process which is analogous to having effectively interacting leads as in a Luttinger liquid.

To carry out such a recombination of the phase fields, since the tunneling only acts at $x = 0$, it is convenient to perform a partial trace in the partition function and integrate out fluctuations in $\phi_c/\phi_f(x)$ for all $x$ away from $x = 0$ [12, 13]. For an Ohmic environment, one can also integrate out the harmonic modes at low frequency [5, 7, 9]. Then, the effective action for the leads and the environment becomes

$$S_{\text{Leads+Env}}^{\text{eff}} = \frac{1}{\beta} \sum_n |\omega_n| \left[ |\phi_c(\omega_n)|^2 + |\phi_f(\omega_n)|^2 + \frac{|\varphi(\omega_n)|^2}{2\omega} \right]$$

(9)
with \( r = R e^2 / h \) the dimensionless impedance of the leads and \( \omega_n = 2 \pi n / \beta \) a Matsubara frequency. In this discrete representation, it is straightforward to combine the phase operator \( \varphi \) and the flavor field \( \phi_f \), in order to maintain canonical commutation relations while doing so, we use the transformation

\[
\phi'_f = \sqrt{g} \left( \phi_f + \frac{1}{\sqrt{2}} \varphi \right), \\
\varphi' = \sqrt{g} \left( \sqrt{\tau} \phi_f - \frac{1}{\sqrt{2}} \varphi \right),
\]

where \( g \equiv 1/(1 + r) \leq 1 \). Now, the effective action for the leads and environment (excluding tunneling) becomes

\[
S_{\text{Leads+Env}}^{\text{eff}} = \frac{1}{\beta} \int \sum_n \left| \omega_n \right| \left( |\phi_c(\omega_n)|^2 + |\phi'_f(\omega_n)|^2 + |\varphi'(\omega_n)|^2 \right),
\]

while the action for the tunneling part reads in the time-domain:

\[
S_T = \int d\tau \left[ \frac{V_S}{\sqrt{2\pi} a_0} e^{-i\frac{\sqrt{2}}{\beta} \phi_c(\tau)} e^{-i\frac{1}{\sqrt{2} \beta} \phi'_f(\tau)} d + \text{c.c.} \\
+ \frac{V_D}{\sqrt{2\pi} a_0} e^{-i\frac{\sqrt{2} \beta}{\sqrt{2} \beta} \phi'_f(\tau)} d + \text{c.c.} \right].
\]

Thus we see that the phase \( \varphi \) has been absorbed into the new flavor field \( \phi'_f \) at the expense of a modified interaction parameter \( g \leq 1 \), while the new phase fluctuation \( \varphi' \) decouples from the system. It turns out that one obtains a very similar effective action by starting from a model of spinless resonant tunneling between Luttinger liquids [12–14], allowing us to obtain the transport properties for arbitrary \( r \) values from previous knowledge accumulated for the Luttinger problem, as discussed in the main text.

### IV. QUANTUM CRITICAL POINT AS MAJORANA STATE

In this final section, we show that the special value \( r = 1 \), corresponding to a fine-tuned circuit impedance \( R = h / e^2 \) (close to our experimental value), allows a detailed understanding of the nature of the quantum critical point. Technically, a standard refermionization [8] of the tunneling term (13), leads to an effective interacting resonant Majorana level.

The mathematical equivalence to the Majorana model starts by performing a unitary transformation [15, 16], \( U = \exp[i(d^d d - 1/2) \phi_c(0) / \sqrt{2}] \), to eliminate the \( \phi_c \) field in the tunneling action, Eq. (13), which now reads:

\[
S_T = \int d\tau \left[ \frac{V_S}{\sqrt{2\pi} a_0} e^{-i\frac{\sqrt{2}}{\beta} \phi'_f(\tau)} d + \frac{V_D}{\sqrt{2\pi} a_0} e^{i\frac{\sqrt{2} \beta}{\sqrt{2} \beta} \phi'_f(\tau)} d \right] + \text{c.c.}
\]

This operation generates a new contact interaction between the dot and the phase field:

\[
H_C = -\pi v_F (d^d d - 1/2) \partial_x \phi_c(x = 0).
\]

For the special value \( g = 1/2 \), we can exactly refermionize the problem by defining fictitious free fermion fields \( \psi_c = e^{i\phi_c/\sqrt{2\pi} a_0} \) and \( \psi_f = e^{i\phi'_f/\sqrt{2\pi} a_0} \) (where we neglect the Klein factors which do not play any role here). Electron waves in the leads and phase fluctuations in the circuit are thus combined into these non-interacting fermionic species. All the complexity of the tunneling process is now encoded in the following expression:

\[
H_{\text{Majorana}} = H_T + H_{\text{Dot}} + H_C
\]

\[
= [V_S \psi_f(0) d + \text{h.c.}] + [V_D \psi_f(0) d + \text{h.c.}]
\]

\[
+ \epsilon_d d^d d - \pi v_F \psi^D_f(0) \psi_c(0) (d^d d - 1/2).
\]

The peculiarity of this effective Hamiltonian is the presence of “pairing” terms, like \( \psi_f(0) d \), in contrast to the initial tunneling term Eq. (2) where the number of the fermions is conserved. This structure motivates the introduction of a Majorana description of the local level, \( d^d = a - ib \) where \( a \) and \( b \) are real fermions satisfying \( \{a, b\} = 0 \) and \( a^2 = b^2 = 1/4 \). The effective Hamiltonian (16) is then readily expressed as

\[
H_{\text{Majorana}} = (V_S - V_D) [\psi_f(0) d - \psi_f(0) a]
\]

\[
+ i(V_S + V_D) [\psi_f(0) d + \psi_f(0) b]
\]

\[
+ 2\pi v_F \psi_f(0) \psi_c(0) ab;
\]

we have, thus, arrived at Eq. (1) of the main text.

A special point of Hamiltonian (17) can then be identified when \( V_S = V_D \) (symmetric tunneling amplitudes to source and drain) and \( \epsilon_d = 0 \) (resonant condition), in which case the \( a \) Majorana does not hybridize to either the leads or the \( b \) Majorana level; the latter is on the other hand tunnel coupled to the fermion bath. In the absence of the density interaction to the field \( \psi_c \) (last term in Eq. (17)), one ends up with a non-interacting Majorana resonant level model, describing a critical state with fractional degeneracy due to the perfect decoupling of the Majorana mode (the ground state entropy is then \( S = 1/2 \log 2 \)). Gogolin and Komnik [16] introduce an extra, fine-tuned Coulomb interaction between the quantum dot and the leads to get rid of the Majorana interaction term. However, this simplifying hypothesis is not appropriate in our case and leads to an incorrect description of the quantum critical point. Indeed, for non-interacting Majoranas, the conductance at low temperature assumes a Fermi liquid behavior with quadratic in \( T \) approach to the unitary value, which corresponds to an exact and unfortunate cancellation of the leading irrelevant operator [14] discussed in the main text. Having a finite Majorana interaction presents two interesting features: (i) the entropy of the critical state remains unchanged at \( S = 1/2 \log 2 \) (the \( a \) mode is not hybridized with the continuum despite the density-density interaction [17–19]); (ii) an anomalous non-Fermi liquid scattering rate is generated [19, 20], yielding a conductance with a linear dependence on temperature and voltage [21]. This recovers the correct behavior due to the leading irrelevant operator in the approach to the critical point [21].
Note that the four-fermion interaction term in Eq. (17) is too large to be quantitatively captured by perturbation theory, but rather drives the Majorana-level model into a strong coupling regime [22] with universal scaling relations describing the quantum critical state in our system. Nevertheless, insight into the striking non-Fermi liquid behavior can be gained by calculating the conductance perturbatively in the interaction [21], as mentioned in the main text. The hybridized Majorana fermion $b$ has a decreasing correlation function at long time, namely $G_{b}(t) = \langle b^{+}(0)b(t) \rangle \sim 1/t$, identical to the behavior of the local (one-dimensional) bath Green’s function $G_{\psi_{r}(x=0)}(t) \propto 1/t$. However the non-hybridized mode $a$ does not decay at long-times, $G_{a}(t) = \langle a^{+}(0)a(t) \rangle \propto 1$. The role of the interaction given by the last term in Eq. (17) is to generate a self-energy for the hybridized mode $b$; at lowest order in perturbation theory, it reads $\Sigma_{b}(t) \propto v_{F}^{2}G_{a}(t)G_{\psi_{c}}^{2}(t) \propto 1/t^{2}$. Now, the current-current correlation function of the original problem turns out to be proportional to the self-energy $\Sigma_{b}$ [21]. Its Fourier transform yields a dependence linear in frequency, and in this way a correction to the unitary conductance which is linear in temperature is produced, $1 - G \propto T^{1}$. Away from the QCP, the Majorana mode $a$ becomes hybridized, and so $\Sigma_{b}$ decays as $1/t^{3}$, which by Fourier transform leads to the expected quadratic-in-energy Fermi liquid behavior.

Finally, for $r$ close to but not exactly 1, corresponding to our experimental situation (the circuit impedance cannot be tuned in situ but rather is fixed by the lithographic process), one ends up with a Majorana model similar to Eq. (17) but with weakly interacting Luttinger leads [23–25] with effective Luttinger parameter $\tilde{g} \propto r$. This residual interaction within the continuum leads to a slight deviation from the linear approach to the unitary conductance, as we indeed observe in our data (see Fig.2a of the main text). Although the critical state is not precisely a Majorana bound state in that case, the associated ground state still possesses entropy $S = \frac{1}{2}\log(1 + r)$ associated with a non-trivial fractional degeneracy [18, 25].