In the format provided by the authors and unedited.
FIG. S1. **Overview of the experimental setup.** At the beam splitter (BS) an amplified 4 fs, 700 nm central wavelength, horizontally polarized pulse train from a laser system with a hollow core fibre compressor is split 65:35% towards a time-of-flight-spectrometer (TOF) arm, and a CEP-meter arm (CEPM), respectively. Fused silica wedges (W) independently control and minimize the dispersion in each arm. The spatial mode of the beam to the experiment is cleaned with a spatial frequency filter (SFF) and its intensity is adjusted by a neutral density filter (ND). The beam is focused inside the ultrahigh vacuum chamber with a 90° off-axis parabolic mirror (OAP, \( f = 50 \text{ mm} \)) on the nanotip. The nanotip is mounted on a xyz-nanopositioning stage that also carries a gas jet (GJ) for Xe measurements and two knife-edges (KE) for focus characterisation. The TOF spectrometer records the flight time of photoelectrons with a microchannel-plate (MCP) that is then converted into kinetic energy. CCD cameras in front (CCD1) and behind the chamber (CCD2) were used for spectral beam profile characterization and the knife-edge measurements, respectively. The MCP and phosphor screen was observed via a mirror and a camera and was used for tip diagnostics.

With all this in place we can now map the phase evolution of the focused pulses with the nanotip. For example, assume that one moves the nanotip to the focus, \((r, z) = (0, 0)\), measures the ATP spectrum, and retrieves the corresponding \(\phi_{co}^{pos}(r=0, z=0)\) value. Then if the nanotip is moved to another position, \((r, z)\), all the laser parameters are kept constant, and nothing is changed in either of the beam paths; then one would expect to measure exactly the same ATP spectrum and resulting \(\phi_{co}^{pos}\) value only with a shift in phase equal to the phase difference between the two points in space, \(\Delta \phi\). In other words,

\[
\phi_{co}^{pos}(r, z) = \phi_{co}^{pos}(0, 0) + \Delta \phi(r, z).
\]

(S4)

In this way one can map the phase evolution, \(\Delta \phi(r, z)\), of the focused laser pulses with respect to the focal point as shown in Figs. 3 and 4, where the uncertainty due to the referencing is also added in quadrature to a sum of \(\approx 0.2 \text{ rad}\).

In practice there are several additional complications. In the example above, all the laser parameters, except the phase, were identical at each point. However, the intensity within the focal region is spatially dependent and the ATP spectrum from the nanotip is intensity dependent. Thus, we adjusted the intensity of the beam going to the nanotip with a neutral density filter for each position while the fraction of the beam going into the CEPM remained untouched to maintain the reference phase. The intensity was adjusted to match the emission rate seen at the focal point and thus ensure that the local intensity at each new nanotip position was the same. We found that the error potentially caused by a small movement of the ND-filter is well below 0.14 rad and hence negligible. Further, to minimize drifts of the relative phase due to ambient fluctuations, periodic measurements at the focal point, \((r, z) = (0, 0)\), were made to account for drifts in the set-up.

In order to determine the focus position in space, the nanotip translation stage also carries two perpendicular knife-edges to perform a measurement of the focal profile, revealing the focus position with a precision of about 50 µm. To further fine tune the tip position, it was moved to the point at which the photoelectron emission was maximized. Finally, the focal point was defined to best symmetrize the recorded phase values \(\phi_{co}^{pos}(0, z)\) as the phase profile is an odd function in \(z\).

To map the focal phase profile in two-dimensions we did not use a cartesian grid. The radial scale of the laser in the focus is defined by the beam width, \(w(z)\). Thus, for each \(z\)-position we chose to follow the hyperbolae at
II. EXPERIMENTAL DETAILS

A schematic of the experimental setup is shown in Fig. S1. For these measurements we used an upgraded commercial laser system ( Femtopower Compact Pro HP/HR™ ) and a hollow-core fibre compressor with a central wavelength tunable from 680 to 750 nm. This system produces a horizontally polarized few-cycle pulse train at 4 kHz with a randomly varying CEP. This beam is split 65:35% between the nanotip chamber and the CEP-meter, respectively, and the dispersion in both arms is controlled and minimized independently with fused silica wedges by optimising on the asymmetry in the ATI-signal of Xe gas, see Ref. 5. With a transform limited pulse duration of 3.7 fs, actual pulse durations of 3.75 ± 0.25 fs are achieved, thus providing practically chirp-free pulses as assumed for the model used6.

The intensity of the beam going to the nanotip chamber is independently controlled using a reflective neutral-density filter. It is also spatially cleaned with an aperture. The beam is focused onto the nanotip with a $f = 50 \text{ mm}, 90^\circ$ off-axis parabolic mirror inside the ultra-high vacuum chamber, hence minimizing the effect of aberrations on the CEP as discussed in Refs. 6, 7. A background pressure of $\sim 10^{-9} \text{ mbar}$ is reached. The effective intensity at the tip’s apex reaches $3.5 \pm 2.0 \times 10^{13} \text{ W/cm}^2$ including the field enhancement at nanometric metal tips8.

The tip is mounted on a 3D-translation stage (SmarAct), which also carries two knife-edges for focus characterisation in $x$- and $y$-directions. The $z$-positioning uncertainty is less than 1 µm while the uncertainty in $r/w(z)$ is less than 0.1, where the radial uncertainty scales with $w(z)$. Note that the translation stage also carries the gas nozzle, which was used to optimize the pulse compression by evaluating the photo-electron emission from Xe.

A microchannel-plate-detector with a phosphor screen is used for tip diagnostics using field emission and field ion microscopy9,10. CCDs in front (CCD1) and behind the chamber (CCD2) are used for spectrally resolved beam profile characterisation and the knife-edge focus size measurements. We obtained an M$^2$ of 1.2 for the beam, hence assuming a TEM$_{00}$ mode as used in Porras’ calculations is a good approximation.

Two different tips were used for the one-dimensional measurements of the phase profile along $z$, which is displayed in Fig. 3. Namely, data sets I and II were made with a tungsten and a gold tip, respectively. The reproducibility of the measurement with two different tips shows the independence of the measurement on the tip.
material. Note that this is expected as the material and tip shape dependent phase retardation effects are predicted to lead only to a phase offset which is eliminated here as the data for each tip is referenced to the phase at the focal point for that tip.

III. FUNDAMENTAL EQUATIONS

For the focus of a Gaussian beam, the intensity in cylindrical coordinates \( r \) (radius) and \( z \) (propagation axis) is given by

\[
I(r, z) = |E(r, z)|^2 = I_0 \left( \frac{w_0}{w(z)} \right)^2 e^{-2\left( \frac{z}{z_R} \right)^2},
\]

with the peak intensity \( I_0 \) and the \( 1/e^2 \) waist radius \( w_0 \) (at \( z = 0 \)). The hyperbola of the local radius is defined as

\[
w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2},
\]

with \( z_R \) being the Rayleigh length:

\[
z_R = \frac{\pi \cdot w_0^2}{\lambda} = \frac{\omega \cdot w_0^2}{2c},
\]

where \( \lambda \) is the wavelength, \( \omega \) the angular frequency and \( c \) the speed of light.

For a focused monochromatic Gaussian beam the local electric field is expressed via

\[
E(r, z) = E_{0,0} \frac{w_0}{w(z)} \cdot e^{-\left( \frac{r}{w(z)} \right)^2} \cdot e^{-ikR(z)} \cdot e^{-i(kz + \Delta \psi(z))},
\]

where \( E_{0,0} \) is the electric field amplitude at \( r = 0, z = 0 \);
\( k = 2\pi/\lambda \) is the wave number; \( R(z) \) is the local radius of curvature and \( \Delta \psi(z) = -\arctan z/z_R \) is the Gouy phase, compare Eq. 1.

Assuming a Gaussian pulse envelope, the temporal evolution of the electric field of laser pulses at a given point in space is commonly described by the pulse duration \( \tau \) (intensity full-width-at-half-maximum), the central angular carrier-frequency \( \omega_0 \), the carrier-envelope phase \( \phi \) (CEP) and the local peak electric field amplitude \( E_{0,r,z} \) as

\[
E(t) = E_{0,r,z} e^{-2\ln 2(t/\tau)^2} \cdot e^{-i(\omega_0 t + \phi)},
\]

which is the derivative of the vector potential \( \partial \mathbf{A}(t)/\partial t \) (compare inset in Fig. S2). The vector potential is important for electron propagation, in particular attosecond streaking where the mapping of time to electron energy is crucial.

Under conditions where chirp is not negligible, the on-axis focal CEP-shift can be amended with a relative chirp term \( C_{r} \), which yields for focussing with a mirror

\[
\Delta \phi(z) = -\arctan \left( \frac{z}{z_R} \right) + \frac{g_0 \cdot \left[ 1 + 2C_{r} \frac{z}{z_R} \right]}{\frac{1}{z_R^2} + \frac{\Delta \phi(z)}{z}}.
\]

IV. INDIRECT METHOD FOR DETERMINATION OF THE SPATIAL PHASE PROFILE

Using a nanotip in the way described and demonstrated here is a direct way to map the phase in the focus of a broadband pulse with unprecedented precision. Additionally, our measurements show that the formulation of the focal phase evolution in terms of the Porras factor is applicable. However, this method is a complex, time consuming, and challenging measurement. For these rea-

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**TABLE S1. Summary of the determined \( g_0 \)-values**

<table>
<thead>
<tr>
<th>Measurement</th>
<th>( g_0 ) from nanotip</th>
<th>( g_0 ) from spectral profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-2.1 ± 0.2</td>
<td>-2.0 ± 1.0</td>
</tr>
<tr>
<td>II</td>
<td>-1.8 ± 0.3</td>
<td>-1.4 ± 0.4</td>
</tr>
<tr>
<td>off-axis</td>
<td>-1.2 ± 0.3</td>
<td>-1.2 ± 0.4</td>
</tr>
</tbody>
</table>
FIG. S5. Stereogram of the CEP focus profile. The CEP measurement off the optical axis from Fig. 4 in stereoscopic/3D format. To watch, mind the following instructions to achieve parallel freeviewing. Size the figure on your reading means to about 11–13 cm width, which corresponds to twice the average human eye separation. Start by holding the graph close to your eyes and ignore the focussing for the moment, just relax the eyes. Look “through” the graph into infinity so that your eyes are parallel. Now slowly move the graph away from your eyes while maintaining the parallel state of your eyes. When the distance approaches about 25–30 cm your eyes should see the graph automatically sharp and three-dimensional.

We have also implemented an indirect method to gain insight into the spatial phase profile of the focus by estimating the Porras factor $g_0$ of the input beam. As seen in Equation 3, the Porras factor $g_0$ depends on the frequency-dependent Rayleigh length of the input beam, $Z_R(\omega)$, which in turn can be calculated from the frequency-dependent input waist. Therefore, if one can determine the frequency-dependent input waist, one can use this for a first-order approximation of the spatial phase profile in the focus.

We have also implemented this technique of using the measured frequency-dependent input beam profile to calculate $Z_R(\omega)$ and the $g_0$-value of our beam geometry concurrently with the three nanotip measurements presented in the text. These examples are shown in Fig. S4 where the Rayleigh length vs. frequency is drawn. As can be seen the Rayleigh lengths are many meters, so estimating the waist by measuring the size of the collimated beam is reasonable. The input beam was measured at the frequencies denoted by the points and fit in first order with a linear function, to derive the Porras factor, $g_0$, which in turn describes the phase profile in the focus. Considering the pioneering character of the method we find fair agreement between the two independent determinations. In the main text, and going in the opposite direction, we have used the phase profile in the focus measured with the nanotip to determine the Porras factor, $g_0$, which in turn reflects the frequency-dependent input beam geometry.

These two directions are summarised by this scheme:

\[
\begin{align*}
\text{Focal CEP-profile (measured by nanotip)} & \Rightarrow \quad \text{Focal CEP-profile (estimated)} \\
\text{Spectral input (calculated)} & \Rightarrow \quad \text{Spectral input (measured)} \\
\text{Focal CEP-profile (measured by nanotip)} & \Rightarrow \quad \text{Focal CEP-profile (estimated)} \\
\end{align*}
\]

The $g_0$ values together with the ones from the nanotip measurement are summarised in Tab. S1. Unlike the nanotip method, this method does not directly measure the spatial phase profile in the focus. Nonetheless, the relative simplicity of the technique is attractive and may be used in the future to infer the focal properties. First, since the beam parameters such as beam radius and beam waist size are typically on the millimetre rather than on the micrometer scale like in the focal region, it is easy to measure them with CCD sensors. Second, one can determine the wavelength-dependence using easily available optical band-pass filters. However, as this is just a first-order approximation and the uncertainties are large, future precision studies of this technique might also show the need to include higher order terms from the expansion of $Z_R(\omega)$, depending on the light source. Further work is clearly needed and warranted to explore the efficacy of this promising technique.


FIG. S6. Projection of Figure 4. The CEP measurement off the optical axis from Fig. 4 projected in the z-r-plane. The contour map is the fit of Eq. 2 and the circles are the measured points.