Specular interband Andreev reflections at van der Waals interfaces between graphene and NbSe$_2$

S1. Interfacial properties of the hBN/bilayer graphene/NbSe$_2$ heterostructure

S1.1 Ohmic electronic interface between BLG and NbSe$_2$

The hBN/BLG/NbSe$_2$ hetero-structures are assembled by fabricating hBN/BLG Hall-bar devices following the standard recipe in [1]. In this process we intentionally do not contact one end of the patterned BLG channel with a Ti/Au electrode, and leave it uncovered for the deposition of NbSe$_2$ (Fig. 1 (b)). The mechanically exfoliated NbSe$_2$ flake is then transferred onto the device in such a way that it creates an overlap region with the BLG from one side, and four gold electrodes from the other side. The so created gold/NbSe$_2$ and BLG/NbSe$_2$ junctions have Ohmic interfacial resistances with typical values of 50-500 $\Omega \mu$m and 1-10 k$\Omega \mu$m respectively (Fig. S1 (a)).

We can estimate the junction resistance $R_{jnc}$ between BLG and NbSe$_2$, with the typical four-probe method [2]. By injecting a source-drain current $I_{sd}=$10 nA across the interface we obtain the resulting voltage difference $V_{sd}$ across the junction (Fig. 1 (b) and inset Fig. S1 (a)). Since one of the voltage probes is connected to the NbSe$_2$ flake while the other voltage probe is connected to the BLG channel, the measured $V_{sd}$ signal, in all generality, contains contributions from the NbSe$_2$ flake, the BLG/NbSe$_2$ interface and the BLG channel. We estimate the interfacial junction resistance of the BLG/NbSe$_2$ interface through the relation $R_{jnc} = (V_{sd} - V_{xx})/I_{sd}$, where we subtract the contribution of the BLG resistance $V_{xx}$ (Fig. S1 (a) inset) from measurements of the voltage difference inside the BLG channel (NbSe$_2$’s m$\Omega$ range resistance can be neglected (Fig. S2 (a))). The so obtained junction resistance provides a more accurate estimate of the voltage drop across the BLG/NbSe$_2$ interface $V_{ns}(V_{bg}) = R_{jnc}(V_{bg})I_{sd}$.

We find that the junction properties can be dramatically improved by applying a source drain current of up to 2mA across the junction, where we obtain $R_{jnc}$ values as low as 400 $\pm$ 200 $\Omega \mu$m. We observe a strong asymmetry in the $R_{jnc}(V_{bg})$ dependence for the electron (positive $V_{bg}$) and hole (negative $V_{bg}$) doped regions (Fig. S1 (a) inset). This behaviour is in good agreement with typical metal/graphene junctions [2-4] and can be explained by pn-junction formation at the junction edge due to work-function matching between the NbSe$_2$ and the BLG. As the resistivity of NbSe$_2$ is several orders of magnitude lower than that of BLG, the majority of the current is injected into the BLG just at the contact edge and not at the contact interface [3]. Here the work-functions $\Phi_{NbSe_2} = 5.9$ eV of NbSe$_2$ and that of the typically used gold $\Phi_{Au} = 4.4$ eV [5] have comparable values.

S1.2. Properties of NbSe$_2$

NbSe$_2$ is a compound from the family of the transitional metal di-chalcogenides (TMDC) [5-7]. Unlike the widely studied semi-conducting TMDC compounds MoS$_2$ and WSe$_2$, NbSe$_2$ is a metal at elevated temperatures and a superconductor below a critical temperature of $T_c \sim 7$K (Fig. S2 (a)). NbSe$_2$ is commonly believed to be a conventional BCS superconductor with a s-wave symmetric Cooper pairing mechanism, and is a type-II superconductor with an upper critical magnetic field of $H_{c2} \sim 4$T. In ultra high purity samples one can obtain a CDW transition below $T_{c,cdw} \sim 33$K that is marked by a slight increase of the sample resistance (Fig. S2 (a)). The interplay of the CDW and the SC ground-states at low temperatures is still a topic of active debate.

The superconducting properties of NbSe$_2$ are very similar to superconductors that are commonly used in prox-
FIG. 2. (a) $R$ vs. $T$ and $B$ of NbSe$_2$. The sample undergoes a CDW phase transition below $T_{cdw} \sim 33$ K and becomes a SC below NbSe$_2$’s $T_c \sim 7$ K. Both, the “hump”-like feature in $R$ at the CDW transition and the sharpness of the SC transition, with a width of $\Delta T \sim 0.1$ K (inset), demonstrate the high purity of the NbSe$_2$ crystal. (b) and (c) $T$- and $B$-dependence of the $G_T/G_{10K}$ vs. $V_{ns}$ traces for $V_{bg} = -50$ V. The characteristic conductance peaks due to the SC proximity effect vanish above the critical temperature $T_c \sim 7$ K and the upper critical magnetic field $H_{c2} \sim 4$ T. Here the peak positions are scaling in agreement with the $T$-dependence expected from the BCS theory $\Delta_{NbSe2}(T) \sim 1.74 \Delta_0 \sqrt{1 - T/T_c}$ (dashed line). The $B$-dependence of the peak positions scales approximately linearly $\Delta_{NbSe2}(B) \propto B$ (dashed line), owing to the type-II SC properties of NbSe$_2$.

S1.3. Superconducting proximity effect at the BLG/NbSe$_2$ interface in the metallic regime

At relatively high carrier densities BLG can be approximated as a good metal. In this regime AR processes at the BLG/NbSe$_2$ interface can be understood by conventional intra-band retro ARs, and manifest itself in a characteristic line-shape of the $G_{1.7K}/G_{10K}(V_{ns})$ traces that are defined by a double-peak structure around zero bias [8]. The voltage positions of these peaks are consistent with the size of NbSe$_2$’s SC gap $\Delta_{NbSe2} \sim 1.2$ meV. By raising the temperature $T$ (Fig. S2 (b)) or the perpendicular magnetic field $B$ (Fig. S2 (c)), these features become continuously smaller and vanish above 7K or 4T, respectively. This is in good agreement with NbSe$_2$’s $T_c$ and $H_{c2}$ values. We can fit the peak positions for various $T$ with the expected temperature dependence for the SC gap from the BCS theory $\Delta_{NbSe2}(T) \sim 1.74 \Delta_0 \sqrt{1 - T/T_c}$ [9]. The approximately linear scaling of $\Delta_{NbSe2}(B)$ is owed to the type-II SC properties of NbSe$_2$.

S2 Theoretical conductance of the BLG/SC interface

S2.1 General formulas

To most accurately resemble the experimental setup we assume a BLG sheet in the normal state in the half plane $x > 0$, and an induced SC gap $\Delta_0$ at $x < 0$. Just as in the experimental system, we assume that the proximity to the SC results not only in the generation of the gap but also in changes of the chemical potential at $x < 0$.

The Bogoliubov-deGennes (BdG) equations describing the electron motion in the system can be written as [10]:

\[
\begin{pmatrix}
\mathcal{H}(\mathbf{K}) + U(x) - \varepsilon_F \\
\Delta^*(x)
\end{pmatrix}
\begin{pmatrix}
u \\
\varepsilon_F - \mathcal{H}(\mathbf{K}) - U(x)
\end{pmatrix}
= \begin{pmatrix}
u \\
\varepsilon
\end{pmatrix}
\]

where $\mathbf{K}$ is the momentum operator and $u$ and $v$ the electron and hole eigenfunctions. Assuming that only the singlet SC is induced and that only electrons of different valleys form Cooper pairs we write the Hamiltonian $\mathcal{H}(\mathbf{K})$ of the normal BLG for one valley as a $4 \times 4$ matrix [11–13]:

\[
\mathcal{H}(\mathbf{K}) = \hbar \omega := 
\begin{pmatrix}
0 & Ke^{i\alpha} & -t_\perp & 0 \\
K^{-1}e^{-i\alpha} & 0 & 0 & 0 \\
-t_\perp & 0 & 0 & Ke^{-i\alpha} \\
0 & 0 & 0 & 0
\end{pmatrix}
\]
$x$-component of $\mathbf{K}$ and $v$ the Fermi velocity.

Due to the SC and the SC gap $\Delta(x)$ the potential $U(x)$ is only finite at $x < 0$ and these quantities can be written in the form:

$$U = \begin{cases} -U_0, & x < 0 \\ 0, & x > 0 \end{cases}$$

$$\Delta = \begin{cases} \Delta_0 e^{i\phi}, & x < 0 \\ 0, & x > 0 \end{cases}$$

The conductance of a SLG/SC junction has first been considered by C.W.J. Beenakker [14, 15], while the same quantity was calculated for BLG in a subsequent publication in by T. Ludwig [16]. However, in the latter work only the limit of $\hbar v_{t\perp} \ll U_0$ was considered. All this does not allow to make a detailed comparison of that theory with the present experimental data.

The sharp change of the potential $U(x)$ and the SC gap $\Delta(x)$ at $x = 0$ is an oversimplification. In the experiment the dependence of these quantities on the coordinates can be rather smooth. Although this may result in a certain overestimation of the transmission amplitude at the interface, we do not think that the dependence of the conductance on $\varepsilon_F$ and $\varepsilon V_{ns}$ can essentially be different in a model with a more smooth potential $U(x)$.

Calculating the conductance one considers scattering of particles having the energy $\varepsilon = \varepsilon V_{ns}$ (counted from $\varepsilon_F$) and moving from right to the left. Here, in accordance with the experiment we assume that $U_0 \gg \varepsilon_F, \Delta_0, \varepsilon V_{ns}$, with the largest energy in being the coupling energy $\hbar v_{t\perp}$ between the layers $\hbar v_{t\perp} \gg U_0$. As the experimental measurements have been presented for $\varepsilon|V_{ns}| < \Delta_0$, we calculate the differential conductance $G_{ns}(\varepsilon)$ for $|\varepsilon| < \Delta_0$. In this region, using a formalism developed by C.J.W. Beenakker[17], one can reduce the calculation of the conductance $G_{ns}(\varepsilon)$ of a N/SC interface to calculation of the transmission $t(\varepsilon)$ and reflection amplitudes $r(\varepsilon)$ for the same interface but in the absence of the SC gap.

According to this theory the differential conductance $G_{ns}(\varepsilon)$ takes the form:

$$G_{ns}(\varepsilon) = 2G_0 Tr \left[ m(\varepsilon)m^+(\varepsilon) \right]$$

where $G_0 = 2e^2/h$ and:

$$m(\varepsilon) = t_{12}(\varepsilon) \left[ 1 - e^{-2i\beta} t_{22}^*(\varepsilon) r_{22}(\varepsilon) \right]^{-1} t_{21}^*(\varepsilon)$$

with:

$$\beta = \arccos(\varepsilon/\Delta_0)$$

Eq. (5-7) show that calculating the transmission $t_{12}(\varepsilon)$, $t_{21}(\varepsilon)$, and reflection $r_{12}(\varepsilon)$, $r_{21}(\varepsilon)$ amplitudes for right and left moving particles at the interface between normal metals one obtains the differential conductance $G_{ns}(\varepsilon)$ of the N/SC interface. The trace $Tr$ over the scattering channels in Eq. (5) reduces in the present model to integration over $k_y$. Overall Eqs. (5-7) demonstrate that in order to calculate the differential conductance $G_{ns}$, one should simply understand the scattering on the interface between two normal metals.

Experimentally, it is convenient to plot not the conductance $G_{ns}(\varepsilon)$ itself but the ratio $G_{ns}(\varepsilon)/G_{nn}(\varepsilon)$, where $G_{nn}(\varepsilon)$ is the conductance of the interface at temperatures exceeding the SC transition temperature $T_c$. $G_{nn}(\varepsilon,\varepsilon_F)$ can be easily expressed in terms of the transmission coefficient as:

$$G_{nn}(\varepsilon,\varepsilon_F) = 2G_0 Tr |t_{12}(\varepsilon,\varepsilon_F)|^2$$

At the same time, the conductance $G_{nn}(\varepsilon,\varepsilon_F)$ given by Eq. (8) is applicable at low temperatures only. In the present experiment the temperature at which the normal conductance is measured exceeds the energies $\varepsilon_F$ and $\Delta_0$. In this case one should take a thermodynamic average $\langle G_{nn}(\varepsilon,\varepsilon_F) \rangle_F$ using the standard formula:

$$\langle G_{nn}(\varepsilon,\varepsilon_F, T) \rangle_F = \frac{1}{4T} \int_{-\infty}^{\infty} G_{nn}(\varepsilon, \mu) \frac{d\mu}{\cosh^2(\frac{\mu - \varepsilon_F}{2T})}$$

Eq. (9) is used in the present work for the calculation of the physical conductance at $T > T_c$. At the same time, the measurements below $T_c$ have been performed at sufficiently low temperature, such that one can use the formulas derived at $T = 0$.

One can see from Eqs. (5, 6) an important difference between the conductance of SLG considered in [14] and that of BLG. In the former case the transmission amplitude is of order one for any $U_0$ due to the so called Klein effect and is equal to a normally incident electron, whereas in the latter case the transmission amplitude decays proportionally to $U_0^{-1/2}$ at large $U_0$ and can be very small. Then, for BLG the conductance $G_{nn}$ is proportional to $|t|^2$, while $G_{ns}$, being proportional to $|t|^4$ can be much smaller than $G_{nn}$.
\begin{align}
E_1 (K) &= \hbar v \left(-t_\perp / 2 - \mathcal{E}(K)\right), \\
E_2 (K) &= \hbar v \left(-t_\perp / 2 + \mathcal{E}(K)\right), \\
E_3 (K) &= \hbar v \left(t_\perp / 2 + \mathcal{E}(K)\right), \\
E_4 (K) &= \hbar v \left(t_\perp / 2 - \mathcal{E}(K)\right)
\end{align}

where:
\[ \mathcal{E}(K) = \sqrt{(t_\perp / 2)^2 + k^2} \]  

Provided that \(t_\perp\) exceeds all the other energies, the bands of the Hamiltonian (2) with the spectra \(E_1 (k)\) and \(E_3 (k)\) are far away from the Fermi surface and their contribution into physical quantities can be neglected. The eigenenergies of the first two low energy bands take in the limit of small \(k_x < t_\perp\) the following form:
\begin{align}
E_2 (k_x) &= \hbar v \left(-t_\perp / 2 + \mathcal{E}(k_x)\right) \approx \frac{\hbar K^2}{t_\perp}, \\
E_4 (k_x) &= \hbar v \left(t_\perp / 2 - \mathcal{E}(k_x)\right) \approx -\frac{\hbar K^2}{t_\perp}
\end{align}

In Eq. (12) \(E_2 (K)\) describes the conduction band (upper part of the spectrum) and \(E_4 (K)\) describes the valence band (lower part of the spectrum). Using the inequality \(\hbar v t_\perp \gg U_0\) we consider only these low lying bands.

Wave functions in the region \(x > 0\).

Starting with the Hamiltonian \(\mathcal{H}(k)\), Eq. (2), and using a diagonalization procedure of [13], we can write out 4-component vectors \(u\) satisfying the equation:
\[(\mathcal{H}(k_x - \varepsilon_F) u = \varepsilon u.\] (13)

We consider the case \(\varepsilon_F > 0\) but the solutions depend on the sign of \(\varepsilon + \varepsilon_F\) and we write them separately for \(\varepsilon + \varepsilon_F > 0\) and \(\varepsilon + \varepsilon_F < 0\). For compact notations we introduce the quantity:
\[K_0 = \sqrt{|\varepsilon + \varepsilon_F|} t_\perp / \hbar v\] (14)

We obtain for the low lying eigenvalues:
\begin{align}
\varepsilon_2 &= -\varepsilon_F + \frac{\hbar v}{} K^2, \\
\varepsilon_4 &= -\varepsilon_F - \frac{\hbar v}{} K^2
\end{align}

In order to calculate the wave functions one should choose an eigenvalue \(\varepsilon\) and determine \(K\) as a function of \(\varepsilon\). It is clear that constructing plain waves in the region of the normal metal \(N_1\) and \(\varepsilon_F > 0\) one should take the solution (15) for \(K\) at \(\varepsilon > -\varepsilon_F\) and of (16) at \(\varepsilon < -\varepsilon_F\). In addition, one has a solution for \(K\) of (16) at \(\varepsilon > -\varepsilon_F\) and of (15) at \(\varepsilon < -\varepsilon_F\). However, the latter solutions are exponentially growing or decaying as functions of \(x\). Nevertheless, they should also be taken into account when matching functions at the interface because of the deep potential \(-U_0\) at \(x < 0\) where the exponential growth can change to plain wave behavior.

Plain wave solutions at \(\varepsilon + \varepsilon_F > 0\). In this region we have left and right moving electrons from the conduction band with the energies from Eq. (15). The solution \(u^R_1\) for right moving particles in this region takes the form:
\[u^R_1 = \frac{e^{i k_x x + i k_y y}}{2 \sqrt{K_0 / t_\perp} \cos \alpha} \left(\begin{array}{c} K_0 / t_\perp e^{-i \alpha} \\ K_0 / t_\perp e^{i \alpha} \end{array}\right),\] (17)

while the solution for the left moving particles \(u^L_1\) reads:
\[u^L_1 = \frac{e^{-i k_x x + i k_y y}}{2 \sqrt{K_0 / t_\perp} \cos \alpha} \left(\begin{array}{c} -K_0 / t_\perp e^{i \alpha} \\ -K_0 / t_\perp e^{-i \alpha} \end{array}\right).\] (18)

The wave functions \(u^R_1\) and \(u^L_1\) correspond to the eigenvalue \(E_2 (k)\) in Eq. (12) and belong to the conduction band. They are normalized assuming the current 1 along the \(x\)-axis for right moving particles and \(-1\) for left moving ones.

Having fixed \(\varepsilon_F > 0\) we have to express \(k_x\) and \(k_y\) in terms of \(K_0\) and \(\alpha\). Here we introduce the angle \(\alpha\) as:
\[k_x - ik_y = K_0 e^{-i \alpha}\] (19)

Then, for \(\varepsilon + \varepsilon_F > 0\) we have the following relations for the variables \(\alpha\) and \(k_x\), with \(\alpha\) varying in the interval \(-\pi/2 < \alpha < \pi/2\):
\[k_x = K_0 \cos \alpha, \quad k_y = K_0 \sin \alpha\] (20)

Plain wave solutions at \(\varepsilon + \varepsilon_F < 0\). For these energies we have plain waves corresponding to right and left moving holes from the valence band in Eq. (16). For the right moving holes we obtain the normalized wave functions:
\[u^R_2 = \frac{e^{i k'_x x + i k'_y y}}{2 \sqrt{K_0 / t_\perp} \cos \alpha'} \left(\begin{array}{c} K_0 / t_\perp e^{-i \alpha'} \\ -K_0 / t_\perp e^{i \alpha'} \end{array}\right),\] (21)
where again the angle $\alpha'$ varies in the interval $-\pi/2 < \alpha' < \pi/2$ : 

$$k'_x = -K_0 \cos \alpha', \quad k'_y = -K_0 \sin \alpha'. \quad (22)$$

Here, the opposite signs in Eq. (20) and Eq. (22) are due to the fact that we now consider holes from the valence band instead of electrons from the conduction band. In general the current of the right moving holes equals to 1, while the current for the left moving equals to $-1$. The solution $u_2^L$ for the left moving particles takes the form :

$$u_2^L = e^{-ik'_x x + ik'_y y} \begin{pmatrix} \frac{-K_0}{t_\perp} e^{i\alpha'} \\ \frac{K_0}{t_\perp} e^{-i\alpha'} \end{pmatrix}. \quad (23)$$

**Decaying and growing solutions at $\varepsilon + \varepsilon_F < 0$.** In addition to the plain waves in Eqs. (17, 18), there are two other solutions $u_1^<$ and $u_1^>$ corresponding to the eigenvalue $\varepsilon_2$ from the conduction band (here as the normalization does not play any role, we omit the pre-factors) :

$$u_1^< = e^{\kappa_x x + i\kappa_y y} \begin{pmatrix} \frac{-iK_0}{t_\perp} e^{i\gamma} \\ \frac{-iK_0}{t_\perp} e^{-i\gamma} \end{pmatrix}. \quad (24)$$

and :

$$u_1^> = e^{-\kappa_x x + i\kappa_y y} \begin{pmatrix} \frac{iK_0}{t_\perp} e^{i\gamma} \\ \frac{iK_0}{t_\perp} e^{-i\gamma} \end{pmatrix}. \quad (25)$$

With the parameters $\kappa_x$ and $\kappa_y$ can be written in the form :

$$\kappa_x = K_0 \cosh \gamma, \quad \kappa_y = K_0 \sinh \gamma \quad (26)$$

**Decaying and growing solutions at $\varepsilon + \varepsilon_F > 0$.** In this region the growing and decaying wave functions correspond to $\varepsilon_4$ in Eq. (16) describing the valence band. We write the growing $u_2^<$ and decaying $u_2^>$ functions as :

$$u_2^< = e^{\kappa'_x x + i\kappa'_y y} \begin{pmatrix} \frac{-iK_0}{t_\perp} e^{i\gamma'} \\ \frac{-iK_0}{t_\perp} e^{-i\gamma'} \end{pmatrix}. \quad (27)$$

and :

$$u_2^> = e^{-\kappa'_x x + i\kappa'_y y} \begin{pmatrix} \frac{-iK_0}{t_\perp} e^{-i\gamma'} \\ \frac{iK_0}{t_\perp} e^{i\gamma'} \end{pmatrix}. \quad (28)$$

The parameters $\kappa'_x$ and $\kappa'_y$ are :

$$\kappa'_x = K_0 \cosh \gamma', \quad \kappa'_y = K_0 \sinh \gamma' \quad (29)$$

Overall the wave functions written in the region $x > 0$ can also be used in the region $x < 0$ after shifting $\varepsilon_F \rightarrow \varepsilon_F + U_0$. We denote these functions by adding the subscript $U_0$, thus obtaining $u_1^{R,L}_U, u_2^{R,L}_U$, etc.

**Transmission $t_{21}$ and reflection $r_{21}$ amplitudes.**

Now we calculate the transmission $t_{21}(\varepsilon)$ and reflection $r_{21}(\varepsilon)$ amplitudes that match the wave functions written on the left and on the right of the interface. Again, we should consider the regions $\varepsilon + \varepsilon_F > 0$ and $\varepsilon + \varepsilon_F < 0$ separately.

**Region $\varepsilon_F < 0$, $\varepsilon + \varepsilon_F > 0$.** The scattering process in $N_2$ includes a plane wave incident from the left, $u_1^{R}_L$, and another one, $u_1^{L} U_0$, reflected from the interface. At the same time, region $N_2$ has an additional solution that is growing with $x$ (decaying from the interface) with the symmetry of $u_1^<$ from Eq. (24). After scattering on the interface of $N_1$ one obtains an outgoing wave with the structure $u_2^R$, Eq. (17), and a decaying wave with the structure $u_2^<$, Eq. (28). We describe the scattering process for $\varepsilon + \varepsilon_F > 0$ and match these functions at the interface :

$$u_1^{R}_L + u_2^< (\varepsilon) + u_1^{L} U_0 + Bu_2^< U_0 = t_{21}(\varepsilon) u_1^{R} + C u_2^>, \quad (30)$$

Eq. (30) presents a system of 4 linear equations. They can be simplified using the fact that $U_0 \gg \varepsilon, \varepsilon_F$ and the amplitudes $t_{21}(\varepsilon)$ and $r_{21}(\varepsilon)$ can be found denoting :

$$\frac{K_0}{K_0 U_0} \approx \sqrt{\frac{\varepsilon + \varepsilon_F}{U_0}} = L(\varepsilon) \ll 1 \quad (31)$$

we obtain using this approximation :

$$t_{21}(\varepsilon) = 2\sqrt{L(\varepsilon) \cos \alpha (1 + \sin^2 \alpha)} \exp(-i\Phi(\alpha)), \quad (32)$$

where the angle $\Phi(\alpha)$ varies in the interval $0 < \Phi(\alpha) < \pi/2$ :

$$\Phi(\alpha) = \arcsin(\sin^2 \alpha). \quad (33)$$
We obtain for the reflection coefficient:

$$r_{22} (\varepsilon) = 1 - 2L (\varepsilon) \sqrt{1 + \sin^2 \alpha} \exp(-i\Phi(\alpha))$$

(34)

with the unitarian condition:

$$|t_{21} (\varepsilon)|^2 + |r_{22} (\varepsilon)|^2 = 1$$

(35)

that immediately follows from Eqs. (32, 34).

**Region \( \varepsilon + \varepsilon_F < 0 \):** In the region \( \varepsilon + \varepsilon_F < 0 \) matching the wave functions at \( x = 0 \) results in the equation:

$$u_{1i}^T + r_{22} (\varepsilon) u_{42}^T + B (\varepsilon) u_{24}^T = t_{21} (\varepsilon) u_2^R + C (\varepsilon) u_4^R,$$

(36)

Using the same approximation as in (31) and proceeding in the same way as for \( \varepsilon + \varepsilon_F > 0 \) one comes to the following results, that again fulfill the unitarian condition in Eq. (35):

$$t_{21} (\varepsilon) = -2\sqrt{L (\varepsilon)} \cos \alpha' \sin \alpha' \exp(i\Phi(\alpha'))$$

(37)

with the reflection amplitude \( r_{22} (\varepsilon) \):

$$r_{22} (\varepsilon) = 1 - 2iL (\varepsilon) \cos \alpha' \exp(i\Phi(\alpha')),$$

(38)

and:

$$\Phi(\alpha') = - \arcsin (\sin^2 \alpha')$$

(39)

It is well known [18] that the transmission amplitude \( t_{12} (\varepsilon) \) is related to the amplitude \( t_{21} (\varepsilon) \) as:

$$t_{12} (\varepsilon) = t_{21} (\varepsilon) e^{i\delta(\varepsilon)},$$

(40)

where no knowledge of the explicit form of the phase \( \delta (\varepsilon) \) is necessary for the calculation of the conductances.

**S2.3 Differential conductance of BLG.**

Using Eqs. (5-8), the calculations of the conductances \( G_{ns} \) and \( G_{nn} \) are straightforward. From the calculated \( t_{21} (\varepsilon) \) and \( r_{22} (\varepsilon) \) one can easily obtain \( t_{21} (-\varepsilon) \) and \( r_{22} (-\varepsilon) \). We concentrate now on the case \( \varepsilon > 0, \varepsilon_F > 0 \) where we can distinguish two regions in the parameter space, where \( \varepsilon < \varepsilon_F \) and the reflections are of the retro type, and \( \varepsilon > \varepsilon_F \) where the reflections are of the specular type.

Explicit expressions for the conductance \( G_{ns} (\varepsilon) \) at \( \varepsilon < \varepsilon_F \) (retro reflection).

Eqs. (5-8) can be rewritten using the integration over \( k_y \) instead of the trace over the transversal channels. However, it is even more convenient to integrate over the incident angle \( \alpha \). The angle \( \alpha \) corresponds to the energy \( \varepsilon \), while another angle \( \alpha' \) corresponds to the energy \( -\varepsilon \). These angles are related to each other by the condition that \( k_y \) is the same in both the cases:

$$\frac{\sin \alpha'}{\sin \alpha} = -\frac{K_0 (\varepsilon)}{K_0 (-\varepsilon)} = -\sqrt{\frac{\varepsilon_F + \varepsilon}{\varepsilon_F - \varepsilon}}$$

(41)

resulting in the condition that Andreev reflections are only possible for angles \( |\alpha| < \alpha_c \), where:

$$\alpha_c = \arcsin \sqrt{\frac{\varepsilon_F - \varepsilon}{\varepsilon_F + \varepsilon}}$$

(42)

With:

$$\Phi(\alpha) = \arcsin (\sin^2 \alpha),$$

(43)

and:

$$\Phi(\alpha') = \arcsin (\sin^2 \alpha') = \arcsin \left[ \frac{(\varepsilon_F + \varepsilon) \sin^2 \alpha}{\varepsilon_F - \varepsilon} \right]$$

(44)

the conductance \( G_{ns} \) can be reduced to the form:

$$G_{ns} = 2G_0 K_0 (\varepsilon) \int_{0}^{\alpha_c} \frac{Y_\varepsilon (\alpha, \alpha') \cos \alpha}{2 |X_\varepsilon (\alpha, \alpha')|^2} d\alpha$$

(45)

where:

$$Y_\varepsilon (\alpha, \alpha') = |t_{21} (\varepsilon)|^2 |t_{21} (-\varepsilon)|^2 = 16L (\varepsilon) L (-\varepsilon) \cos \alpha \cos \alpha' \left( 1 + \sin^2 \alpha \right) \left( 1 + \sin^2 \alpha' \right)$$

(46)

and:

$$X_\varepsilon (\alpha, \alpha') = \frac{1}{2} \left[ 1 - e^{-2i\beta} r_{22}^* (-\varepsilon) r_{22} (\varepsilon) \right]$$

(47)

Here we can write \( |X_\varepsilon (\alpha, \alpha')|^2 \) entering Eq. (46) only up to quadratic terms in \( L (\varepsilon) \):

$$|X_\varepsilon (\alpha, \alpha')|^2 = \sin^2 \beta - 2 \sin^2 \beta \left( 1 + \sin^2 \alpha \sin (\beta + \Phi(\alpha)) \right) + \left( 1 + \sin^2 \alpha' \sin (\beta - \Phi(\alpha')) \right) + L (-\varepsilon) \sqrt{1 + \sin^2 \alpha' \sin (\beta - \Phi(\alpha'))} + L (\varepsilon) \sqrt{1 + \sin^2 \alpha' \sin (\beta + \Phi(\alpha'))} + 2L (\varepsilon) L (-\varepsilon) \sqrt{1 + \sin^2 \alpha' \sin (\beta + \Phi(\alpha'))} \times \left[ \cos (2\beta - \Phi(\alpha')) + \Phi(\alpha) - 2 \sin \Phi(\alpha) \sin \Phi(\alpha') \right]$$

(48)
**Explicit expressions for the conductance** \( G_{ns}(\varepsilon) \) **at** \( \varepsilon > \varepsilon_F \) **(specular reflection).**

Here the angles \( \alpha \) and \( \alpha' \) are related to each other as:

\[
\sin \alpha' \sin \alpha = \frac{K_0(\varepsilon)}{K_0(-\varepsilon)} = \frac{\sqrt{\varepsilon + \varepsilon_F}}{\varepsilon - \varepsilon_F} \tag{49}
\]

and the critical angle \( \alpha_c \) equals:

\[
\alpha_c = \arcsin \sqrt{\frac{\varepsilon - \varepsilon_F}{\varepsilon + \varepsilon_F}} \tag{50}
\]

For \( \Phi(\alpha) \) we use Eq. (43) and obtain for \( \Phi(\alpha') \):

\[
\Phi(\alpha') = \arcsin \left( \frac{\varepsilon + \varepsilon_F}{\varepsilon - \varepsilon_F} \sin^2 \alpha \right) \tag{51}
\]

**Conductance for** \( \varepsilon > \varepsilon_F \). In this case the conductance \( G_{ns}(\varepsilon) \) is determined by Eq. (45). Therefore we obtain:

\[
Y_{\varepsilon}(\alpha, \alpha') = |t_{21}(\varepsilon)|^2 |t_{21}(-\varepsilon)|^2 = 16L(\varepsilon) L(-\varepsilon) \cos \alpha (1 + \sin^2 \alpha) \cos \alpha' \sin^2 \alpha' \tag{52}
\]

and as \( X_{\varepsilon}(\alpha, \alpha') \) is determined by Eq. (47) we get:

\[
|X_{\varepsilon}(\alpha, \alpha')|^2 = \sin^2 \beta - 2 \sin \beta \left[ L(\varepsilon) \sqrt{1 + \sin^2 \alpha \sin (\beta + \Phi(\alpha))} - L(-\varepsilon) \cos \alpha' \cos (\beta + \Phi(\alpha')) \right] + L^2(\varepsilon) (1 + \sin^2 \alpha) + L^2(-\varepsilon) \cos \alpha' \sqrt{1 + \sin^2 \alpha} \times \left[ \sin (2\beta + \Phi(\alpha) + \Phi(\alpha')) - 2 \cos \phi(\alpha) \sin \Phi(\alpha') \right] \tag{53}
\]

**Conductance** \( G_{nn} \) **of the interface between two normal metals.**

The transmission amplitude \( t_{12}(\varepsilon) \) determines the conductance \( G_{nn} \) between two normal metals. At a fixed \( \varepsilon_F \) the conductance \( G_{nn}(\varepsilon) \) is given by the following formula:

\[
G_{nn}(\varepsilon) = G_0 K_0(\varepsilon) \int_{-\pi/2}^{\pi/2} |t_{12}(\varepsilon)|^2 \cos \alpha d\alpha \tag{54}
\]

which with Eq. (32) results in:

\[
G_{nn}(\varepsilon) = G_0 K_0(\varepsilon) \int_{-\pi/2}^{\pi/2} 4L(\varepsilon) \cos^2 \alpha (1 + \sin^2 \alpha) d\alpha = \frac{5\pi}{2} G_0 K_0(\varepsilon) L(\varepsilon) \tag{55}
\]

The conductance between of the normal metals can conveniently normalize the conductance \( G_{ns} \) and allows to compare with the experimental quantities:

\[
\tilde{G}_{ns}(\varepsilon) = \frac{G_{ns}(\varepsilon)}{(G_{nn}(\varepsilon, \varepsilon_F, T))_F} \tag{56}
\]

It is important to emphasize that, although the conductance \( G_{ns}(\varepsilon) \) is symmetric with respect to the inversion \( \varepsilon \rightarrow -\varepsilon \), the normal conductance \( G_{ns} \) is not. Therefore, the reduced conductance \( \tilde{G}_{ns}(\varepsilon) \) is not symmetric either.

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