Collective non-perturbative coupling of 2D electrons with high-quality-factor terahertz cavity photons

Qi Zhang, Minhan Lou, Xinwei Li, John L. Reno, Wei Pan, John D. Watson, Michael J. Manfra, and Junichiro Kono*

*kono@rice.edu

1 GaAs quantum wells

We used two GaAs quantum well (QW) samples that were grown by molecular beam epitaxy (MBE) with Si δ-doping. Sample 1 contained a 30-nm-thick single QW, and the electron density and mobility were $3 \times 10^{11}$ cm$^{-2}$ and $4 \times 10^6$ cm$^2$/Vs, respectively. Sample 2 had a similar structure, and its density and mobility were $5 \times 10^{10}$ cm$^{-2}$ and $4.4 \times 10^6$ cm$^2$/Vs, respectively.

The electron density of a 2DEG is sensitive to light illumination at helium temperatures. Electrons are able to jump from the Si δ-doping layer into the QW by absorbing photons. By changing the illumination time, we were able to tune the QW electron density within a certain range, depending on the sample. The density was monitored by in-situ transport measurements. To erase the effect of illumination, samples were slowly warmed up to a temperature higher than 200 K.

To remove the bulk GaAs substrate, we utilized a mixture solution of citric acid (1 g C$_6$H$_8$O$_7$: 1 mg DI H$_2$O) with hydrogen peroxide (30% H$_2$O$_2$ and 70% H$_2$O by volume) to selectively etch away the GaAs substrate [1]. The optimized volume ratio between citric acid solution and hydrogen peroxide was 5:1.

2 Time-domain THz magnetospectroscopy

We used the methods of polarization-resolved time-domain terahertz (THz) magnetospectroscopy [2–6] to study the 2DEG samples in cavities a split-coil superconducting magnet. Our laser source was a Ti:sapphire regenerative amplifier (Clark-MXR, Inc., CPA-2001) with 775 nm center wavelength, 1 kHz repetition rate, and 200 fs pulse width. The beam was split into two, one for THz generation, and the other for detection. The nonlinear crystals used for both generation
and detection were 1-mm-thick \( \langle 110 \rangle \)-oriented zinc telluride (ZnTe). The THz pulse duration was about 2 ps, with a bandwidth from 0.2 to 2.6 THz. The peak electric field strength was on the order of 100 V/cm at the sample.

Figure S1: (Top) experimental configuration and (bottom) typical polarization-resolved transmission data showing coherent cyclotron oscillations.

The top picture in Fig. S1 shows the experimental configuration. The incident beam was linearly polarized by the first polarizer, and by rotating the second polarizer, the transmitted THz field was measured in both the \( x \) and \( y \) directions. The bottom graph in Fig. S1 shows a typical transmitted THz waveform in the time domain. Each blue dot represents the tip of the THz electric field, \( \mathbf{E} = (E_x, E_y) \), at a given time. The red traces are the projections of the waveforms onto the \( E_x-t \) plane and \( E_x-E_y \) plane. This waveform is the difference between two traces taken at 0 T and 2.5 T, \( E_{0T}(t) - E_{2.5T}(t) \), which is proportional to the THz-induced current at the CR frequency of the 2DEG [6].
3 THz 1D photonic-crystal cavities

A perfect 1D photonic crystal exhibits band gaps (or stop-bands) in its transmission spectrum due to resonant Bragg reflections. If the photonic crystal has a defect inside, a THz wave will be trapped by the defect, which leads to a transmission mode inside the band gaps. Different from its near-infrared and visible counterparts, a THz 1D photonic-crystal cavity (PCC) can utilize dielectric thin slabs and air as the high and low index materials, respectively. The air-Si combination provides a large index contrast and thus can significantly reduce the number of layers needed on each side of the cavity [7, 8]. The 1D PCC is also a quite feasible design for our transmission THz time-domain spectroscopy system to study strong THz-matter coupling in thin film samples. It is straightforward to place the thin film at the electric or magnetic field maximum for electronic or magnetic excitations, respectively.

We used commercial thin silicon wafers, with a refractive index of 3.4, as the high index material. Air spacing was created by thin copper films with holes in the middle. For the central layer (“defect”), we used Si (Cavity 1) and sapphire (Cavity 2). The central layer was twice thicker than the others. A thin 2DEG film was transferred onto one surface of the center layer, where the electric field maximum is located. Figure S2a shows an experimental transmission spectrum measured for Cavity 1. Within the ~2.5 THz bandwidth, the power transmission spectrum shows three transmission stop-bands, i.e., photonic band gaps. At the center of each stop-band is a sharp cavity mode. The full-width-at-half-maximum (FWHM) of the cavity mode was calculated to be 2.3 GHz, 2.8 GHz, and 2.6 GHz for the first, second, and third modes, respectively. The theoretically expected linewidth was around 1 GHz for a perfectly aligned cavity. The small discrepancy between experiment and theory is likely due to the imperfect parallelism of Si wafers and copper thin films. The highest $Q$ achieved in this structure was 810 for the third cavity mode. Technically, in time-domain spectroscopy, a time window of $\Delta T$ provides a frequency resolution of $1/\Delta T$. We performed a 500 ps time window scan in the measurement shown above, which means that the linewidths of the cavity modes were not instrument-limited. The $Q$-factor can be further increased by using three layers of silicon on each side of the central defect layer.
Figure S2: a, Experimental transmission spectrum of Cavity 1 without a 2DEG. b, c, and d show spectra (red open circles) for, respectively, the first, second, and third cavity modes together with Lorentzian fits as solid lines. The FWHM of the three modes were 2.3 GHz, 2.8 GHz, and 2.6 GHz, respectively.

4 Strong coupling of cyclotron resonance with cavity modes in Cavities 1 and 2: Additional data

Strong coupling between CR and the second and third cavity modes is shown in Fig. S3, a and b. As in the first mode, CR splits into two peaks, the lower polariton (LP) and upper polariton (UP) branches. The vacuum Rabi splitting values of the second and third modes were 66 ± 1 GHz and 60 ± 1 GHz, respectively. These values are slightly smaller than the vacuum Rabi splitting observed for the first mode (74 GHz, see Table 1), while the polariton linewidth is larger, especially for the third mode, likely due to imperfect surface contact between the 2DEG and the central silicon wafer. The higher-order modes are more susceptible to imperfec-
Figure S2: a, Experimental transmission spectrum of Cavity 1 without a 2DEG. b, c, and d show spectra (red open circles) for, respectively, the first, second, and third cavity modes together with Lorentzian fits as solid lines. The FWHM of the three modes were 2.3 GHz, 2.8 GHz, and 2.6 GHz, respectively.

4 Strong coupling of cyclotron resonance with cavity modes in Cavities 1 and 2: Additional data

Strong coupling between CR and the second and third cavity modes is shown in Fig. S3, a and b. As in the first mode, CR splits into two peaks, the lower polariton (LP) and upper polariton (UP) branches. The vacuum Rabi splitting values of the second and third modes were $66 \pm 1$ GHz and $60 \pm 1$ GHz, respectively. These values are slightly smaller than the vacuum Rabi splitting observed for the first mode (74 GHz, see Table 1), while the polariton linewidth is larger, especially for the third mode, likely due to imperfect surface contact between the 2DEG and the central silicon wafer. The higher-order modes are more susceptible to imperfect contact.

Figure S3: Anticrossing of CR with a, the second cavity mode and b, the third cavity mode in Cavity 1. In a, the magnetic field is increased from 2.4 T (bottom) to 3.4 T (top). In b, the magnetic field is increased from 4.6 T (bottom) to 5.2 T (top). c, Summary of the measured peak positions of the lower and upper polariton branches for the three cavity modes in Cavity 1, the $\nu$ value indicates the Landau-level filling factor, $n_e h/eB$, at each resonance.
tions due to their shorter wavelengths; hence, we believe that the polariton modes are inhomogeneously broadened in the second and third cavity modes. The peak positions for the LP and UP modes are presented in Fig. S3c.

Cavity 2 showed a larger vacuum Rabi splitting of 90 GHz for both the first and second cavity modes (see Fig. S4), due to a better 2DEG-sapphire interface. However, the linewidths of the cavity modes and polaritons in Cavity 2 data were instrument-limited because of the insufficient scan time range used (80 ps).

Figure S4: Anticrossing of CR with the a first and b second mode in Cavity 2. All traces are vertically offset for clarity. In a, the magnetic field is increased from 0.5 T (bottom) to 1.35 T (top) with a step size of 0.05 T. In b, the field is increased from 2.1 T (bottom) to 3.2 T (top) with a step size of 0.05 T. The linewidth of all peaks was instrument limited because of the insufficient scan time range used in these measurements (80 ps). c, Peak positions for the LP and UP branches for the first (blue) and second (red) cavity modes.
5 Rabi oscillations in the time domain

Figure S5: a, Vacuum Rabi oscillations in the time domain. CR is resonantly coupled with the second cavity mode at 2.975 T in Cavity 1. $\Delta E_y = E_y(+2.975\, \text{T}) - E_y(-2.975\, \text{T})$ is the measured difference between the transmitted THz waveforms recorded at +2.975 T and −2.975 T in the $y$-polarization direction, as shown by the top black trace. More clear vacuum Rabi oscillations can be observed by removing the central cavity mode due to the CR inactive THz component with a numerical notch filter (bottom blue trace). The beating nodes of the LP and UP modes are indicated by arrows. b, Frequency-domain spectra for $\Delta E_y$ before and after the numerical removal of the cavity mode.

In the strong-coupling regime, the most pronounced feature of light-matter interaction is the coherent repetitive energy transfer between the matter resonance and the cavity photons as observed in atomic gases [9], semiconductor QW-exciton-polaritons [10–13], and spin ensemble systems [14]. Because THz time-domain magnetospectroscopy is an ultrafast time-resolved technique, we are naturally able to see vacuum Rabi oscillations directly in the time domain. Experimentally, to minimize the background and only probe the circularly-polarized THz fields, we measured the $y$-polarization component, $E_y$, of the transmitted THz wave, while the incident THz wave was fully $x$-polarized. We measured $E_y$ in positive ($+B$) and negative ($−B$) fields, and took the difference $\Delta E_y = E_y(+B) - E_y(-B)$, to further eliminate the background. With this method, $\Delta E_y$
only came from the circularly polarized LP and UP modes and the CR inactive mode with the counter-circular polarization. As such, the measured $\Delta E_y$ signal showed strong beating between the two polariton modes and CR inactive modes, as shown in Fig. S5, top trace. By numerically filtering out the center cavity mode, clear beating between the two polaritons alone can be resolved.

### 6 Quantum mechanical simulations

The Jaynes-Cummings (JC) model [15] is widely used in describing light-matter interactions in two-level atoms or atomic-like two-level systems interacting with a quantized light field. The JC Hamiltonian for a two-level system with energy separation $\hbar\omega_{\text{atom}}$ interacting with cavity photons with mode frequency $\omega_0$ reads

$$H_{JC} = \hat{H}_{\text{atom}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{int}} \quad (S1)$$

$$\hat{H}_{\text{atom}} = \frac{1}{2}\hbar\omega_{\text{atom}}\hat{\sigma}_z \quad (S2)$$

$$\hat{H}_{\text{cavity}} = \hbar\omega_0\hat{a}^\dagger\hat{a} \quad (S3)$$

$$\hat{H}_{\text{int}} = \hbar g (\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a}) \quad (S4)$$

where $\hat{\sigma}_\pm = \hat{\sigma}_x \pm i\hat{\sigma}_y$ are the raising and lowering operators for the two-level system, $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$ are the Pauli matrices of the two-level system, and $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators for the cavity photons. $\hat{H}_{\text{int}}$ is the light-matter interaction term, where the coupling strength is represented by $g$. Using the JC Hamiltonian, one can show that the Rabi splitting

$$\Omega_R = \sqrt{\Delta^2 + 4g^2(n_{\text{ph}} + 1)}, \quad (S5)$$

where $\Delta = \omega_{\text{atom}} - \omega_0$ is the detuning and $n_{\text{ph}}$ is the averaged number of photons inside the cavity. Even when the average number of photons inside the cavity is zero ($n_{\text{ph}} = 0$), one can see that the atoms are still coupled with cavity photons, i.e., the vacuum field through quantum fluctuations. In this case, the Rabi splitting at zero detuning and zero average number of photons

$$\Omega_R(\Delta = 0, n_{\text{ph}} = 0) = 2g \quad (S6)$$

is known as the vacuum Rabi splitting.

One assumption made in the JC model is the rotating wave approximation (RWA), which neglects the other two terms (proportional to $\hat{a}\hat{\sigma}_-$ and $\hat{a}^\dagger\hat{\sigma}_+$), known
as the counter-rotating terms, in $\hat{H}_{\text{int}}$, which is valid when $g \ll \omega_0$. This condition is generally satisfied if the system is far from the ultrastrong-coupling regime. However, the JC model is not expected to be able to describe our CR-cavity system well for two reasons. First, a Landau-quantized 2DEG is not a two-level system in general; rather, it is an ensemble of many harmonic oscillators. Second, the large dipole moment of CR provides strong light-matter coupling. It has been shown that $g$ can go beyond $0.1\omega_0$ [16, 17], where the use of the RWA is not well justified. Hence, both the counter-rotating terms and the diamagnetic term need to be included to describe our system. We therefore used a more accurate Hamiltonian, which is written in the following way [18]:

$$\hat{H}_{\text{tot}} = \hat{H}_{\text{CR}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{int}} + \hat{H}_{\text{dia}}$$  \hspace{1cm} (S7)

$$\hat{H}_{\text{CR}} = \hbar\omega_e \hat{b}^{\dagger} \hat{b}$$  \hspace{1cm} (S8)

$$\hat{H}_{\text{cavity}} = \hbar\omega_0 \hat{a}^{\dagger} \hat{a}$$  \hspace{1cm} (S9)

$$\hat{H}_{\text{int}} = \hbar g \hat{a}^{\dagger} \hat{b} - \hat{b}^{\dagger} \hat{a} + \hbar g \hat{a}^{\dagger} \hat{b}^{\dagger} - \hat{b}$$  \hspace{1cm} (S10)

$$\hat{H}_{\text{dia}} = \frac{\hbar g^2}{\omega_c} (\hat{a}^{\dagger} \hat{a}^{\dagger} + \hat{a} \hat{a}^{\dagger} + \hat{a}^{\dagger} \hat{a} + \hat{a} \hat{a})$$  \hspace{1cm} (S11)

Here, $\hat{H}_{\text{CR}}$ describes the energy of the 2DEG in a magnetic field ($B$) with frequency $\omega_c = eB/m^*$, $m^* = 0.07m_e$ is the electron effective mass of GaAs, $m_e = 9.11 \times 10^{-11}$ kg, and $\hat{b} (\hat{b}^{\dagger})$ is the annihilation (creation) operator for collective CR excitations. $\hat{H}_{\text{cavity}}$ is for describing cavity photons with mode frequency $\omega_0$. The coupling strength between CR and a cavity photon is $g$. The last term in $\hat{H}_{\text{tot}}$ is the diamagnetic term; mathematically, it is the quadratic term of the vector potential $A$ of the light field, i.e., the so-called $A^2$ terms. The pre-factor $\hbar g^2/\omega_c$ suggests that the $A^2$ term is negligibly small in the weak-coupling regime but may have measurable effects or can even become dominant in the strong and ultrastrong-coupling regimes.

The magnitude of the vacuum Rabi splitting, $2g$, is determined by the CR dipole moment of an individual electron ($d_{\text{CR}}$), the strength of the vacuum field in the cavity ($E_{\text{vac}}$), and the total number of electrons ($N_e$). $2g$ for a single quantum well can be written as [18]

$$2g = 2d_{\text{CR}} \cdot E_{\text{vac}} \cdot \sqrt{N_e} = \sqrt{\frac{2e^2 \omega_c n_e \omega_0}{\varepsilon_r m^* L_z}}$$  \hspace{1cm} (S12)

where $n_e$ is the 2D electron density, $\varepsilon_r$ is the dielectric constant of the medium inside the cavity (13 for GaAs), and $L_z$ is the effective cavity length. In our THz
PCC cavity, the $E_{\text{vac}}$ is calculated to be on the order of $10^{-3}$ V/cm. In our experiments, the actual electric field of the THz probe at the cavity frequency was varied between 0.005 and 0.1 V/cm. The corresponding photon number was between 10 and $10^4$. The reason we still observed vacuum Rabi splitting is because a GaAs 2DEG in a magnetic field in an electromagnetic field is a linear system (à la, a harmonically driven simple harmonic oscillator). Different from two-level atoms, the coupling strength does not depend on the driving field strength. Hence, the Rabi splitting in a cavity-2DEG system does not have a square root dependence on the cavity photon number. This is confirmed by the THz field strength dependence of polariton positions shown in Fig. S6, in which the THz probe field was attenuated to the $10^{-3}$ V/cm level, approaching the vacuum field value, and yet, the polariton positions remained unchanged.

Figure S6: The probe field strength dependence of the polariton splitting. a, The peak electric field was 5, 10, 20, 30, and 45 V/cm (from bottom to top trace). The corresponding THz field at the cavity frequency was $\sim 10^{-3}$ times smaller than the peak field. b, The corresponding polariton splitting for each case. The splitting remained unchanged from the case of $10^3$ average cavity photons to the quantum limit of only a few photons.
PCC cavity, the electric field is calculated to be on the order of $10^{-3}$ V/cm. In our experiments, the actual electric field of the THz probe at the cavity frequency was varied between 0.005 and 0.1 V/cm. The corresponding photon number was between $10^2$ and $10^4$. The reason we still observed vacuum Rabi splitting is because a GaAs 2DEG in a magnetic field in an electromagnetic field is a linear system (a linearly driven simple harmonic oscillator). Different from two-level atoms, the coupling strength does not depend on the driving field strength. Hence, the Rabi splitting in a cavity-2DEG system does not have a square root dependence on the cavity photon number. This is confirmed by the THz field strength dependence of polariton positions shown in Fig. S6, in which the THz probe field was attenuated to the $10^{-3}$ V/cm level, approaching the vacuum field value, and yet, the polariton positions remained unchanged.

$\begin{align*}
\text{Figure S6: The probe field strength dependence of the polariton splitting.}
\end{align*}$

$\begin{align*}
a, \text{ The peak electric field was 5, 10, 20, 30, and 45 V/cm (from bottom to top trace). The corresponding THz field at the cavity frequency was } \sim 10^{-3} \text{ times smaller than the peak field.}
b, \text{ The corresponding polariton splitting for each case. The splitting remained unchanged from the case of } 10^{-3} \text{ average cavity photons to the quantum limit of only a few photons.}
\end{align*}$

$\begin{align*}
\text{Figure S7: a and b present the best fits to our experimental data of the two cavities using three different Hamiltonians: the full CR-cavity Hamiltonian (black solid), the full CR Hamiltonian without the } A^2 \text{ term (black dashed), and the Jaynes-Cummings Hamiltonian (blue dotted). Only the full CR-cavity Hamiltonian reproduced the experimental data well. c and d show the normalized fitting deviation as a function of } g/\omega_0 \text{ ratio. The best fit was achieved when } g/\omega_0 = 0.09 \text{ for Cavity 1 and 0.12 for Cavity 2.}
\end{align*}$
The quadratic form of $\hat{H}_{\text{tot}}$ makes exact diagonalization possible through the generalized Hopfield method [19], which is in essence a Bogoliubov transformation of the bare CR and cavity photon operators. The operators of the new normal modes, the polaritons, are linear combinations of $\hat{a}$, $\hat{a}^\dagger$, $\hat{b}$, and $\hat{b}^\dagger$. The Hopfield transformation matrix is the following [18, 19]:

$$M = \begin{pmatrix} \omega_0 + \frac{2g^2}{\omega_c} & ig & -\frac{2g^2}{\omega_c} & ig \\ -ig & \omega_c & ig & 0 \\ \frac{2g^2}{\omega_c} & ig & -\omega_0 - \frac{2g^2}{\omega_c} & ig \\ ig & 0 & -ig & -\omega_c \end{pmatrix}.$$  \hspace{1cm} (S13)

By diagonalizing $M$, one is able to obtain the frequencies of polaritons from the eigenvalues. With the experimental cavity frequency and effective mass values, we varied $g$ to fit the experimental data. The best fit was determined by minimizing the standard deviation between fitting curves and the measured points. Figure S7, a and b, present the best fits with three different Hamiltonians for both cavities. Figure S7, c and d, show the normalized standard deviations of fitting results as a function of $g/\omega_0$ for both Cavities 1 and 2. At the minimum deviation ($<1\%$), $g/\omega_0$ is 0.09 for Cavity 1 and 0.12 for Cavity 2. The full CR-cavity Hamiltonian provides the best fit, while the other two fail to show the non-negligible blue shift of the polariton modes.

7 Classical electrodynamic simulation

The strong coupling of CR and THz cavity fields can be described by classical electrodynamics as well. Rabi splitting itself is not a purely quantum mechanical effect. In the classical picture, vacuum Rabi splitting can be explained as normal mode coupling between CR and cavity modes. That is, the upper and lower polaritons can be viewed as the normal modes of the coupled system. To reproduce experimental features by classical electromagnetic simulations, we utilized the transfer matrix method, which is a powerful tool for describing optical properties of multilayered structures, to calculate the transmission spectra of our 2DEG-THz cavity structure.

The complex refractive index of a 2DEG thin film can be obtained through the Drude model. The elements of the Drude conductivity tensor of a 2DEG in a
magnetic field applied in the $z$ direction are given by

$$\sigma_{xx} = \frac{\sigma_0 \omega \tau}{(1 - i\omega \tau)^2 + (\omega_c \tau)^2}, \quad \sigma_{xy} = \frac{\sigma_0 \omega_c \tau}{(1 - i\omega \tau)^2 + (\omega_c \tau)^2},$$

(S14)

$$\sigma_{CRA} = \sigma_{xx} + i \sigma_{xy}, \quad \sigma_{CRI} = \sigma_{xx} - i \sigma_{xy},$$

(S15)

where $\sigma_0 = e n_e \mu_e$ is the DC conductivity, $\mu_e = e \tau / m^*$ is the electron mobility, $\tau$ is the momentum scattering time, $\sigma_{xx}$ and $\sigma_{xy}$ are the longitudinal and Hall conductivities, respectively, and $\sigma_{CRA}$ and $\sigma_{CRI}$ are the conductivities for the CR active and CR inactive modes, respectively.

Figure S8 shows good agreement between experimentally measured (red solid curve) and theoretically calculated (blue dashed curve) transmission spectra for the CR active mode. The transfer matrix method was used for the calculation. The actual density and mobility values of the 2DEG were used in the calculation. The calculated normal mode splitting is 86 GHz for all three cavity modes. This value is similar to the experimental value of 74 GHz for the first mode.

For Cavity 2, excellent agreement between calculation and experiments was achieved; see Fig. S9. The peak positions of the polaritons and the cavity mode were well reproduced by our classical model. In contrast to the ultrasharp polaritons observed within the photonic stop-bands, a much broader CR linewidth is observed outside the stop-bands due to the unsuppressed superradiance decay, as shown in Fig. S9b; interestingly, this feature is reproduced by the classical model, as shown in Fig. S9a, indicating that superradiance [20] (i.e., ‘radiation damping’ [21]) is not a purely quantum mechanical concept.

© 2016 Macmillan Publishers Limited, part of Springer Nature. All rights reserved.
Figure S8: a, Experimental transmission spectrum of Cavity 1 is reproduced by the transfer matrix method at 0 T. b, Calculated cavity power transmittance for a CR active THz wave with realistic values of 2DEG density ($3 \times 10^{11}$ cm$^{-2}$) and mobility ($4 \times 10^6$ cm$^2$/Vs) in magnetic fields up to 6 T. The calculation matches the experimental peak positions (yellow open circles) of the polaritons in Cavity 1.
Figure S9: a, Calculated cavity power transmittance for linearly polarized THz probe pulse with a realistic 2DEG density for Cavity 2. b, Experimental transmittance of Cavity 2 from 0 T to 3.5 T with a color scale from 0 to 1 in transmittance. c, Same experimental data with a color scale from 0 to 0.06 in transmittance. The polariton and cavity modes are well resolved. They match the calculated spectrum in (A) well. The linewidths of the cavity mode and polaritons are instrument limited by insufficient time scan range used (80 ps).
Finally, it should be noted that the red-shift of the cavity mode with increasing magnetic field can be well reproduced by the transfer matrix method simulation, as shown in Fig. S10. At a given positive magnetic field direction, only one circular polarization, the CR-active (CRA) mode, interacts with the 2DEG; the other mode, the CR-inactive (CRI) mode, is prohibited from interacting with the 2DEG due to the CR selection rule. On the other hand, the CRI mode at a positive magnetic field (+B) is equivalent to the CRA mode at a negative magnetic field (−B). As shown in Fig. S10a, the residual cavity mode corresponding to the CRI circular polarization component (blue peak) exhibits a small red-shift with increasing magnetic field. After folding that branch to the negative magnetic field region, it falls nicely on the CRA-UP branch, as presented in Fig. 3b and c. As explained in the main text, this red-shift is enable by the unusually strong coupling in our system for the detuning ∆ > ω0. From an alternative viewpoint, the residual cavity mode is actually the upper polariton branch of CRI (CRI-UP), and the red-shift is a result of anti-crossing between CR and the THz cavity mode at −1 Tesla, as shown in Fig. S10c. In our experiments, a linearly polarized THz probe beam was used, which resulted in a 50%-50% superposition the CRA and CRI features. Experimental polariton peak positions match well with the classical electromagnetic field simulation, as presented in Fig. S10d.
Finally, it should be noted that the red-shift of the cavity mode with increasing magnetic field can be well reproduced by the transfer matrix method simulation, as shown in Fig. S10. At a given positive magnetic field direction, only one circular polarization, the CR-active (CRA) mode, interacts with the 2DEG; the other mode, the CR-inactive (CRI) mode, is prohibited from interacting with the 2DEG due to the CR selection rule. On the other hand, the CRI mode at a positive magnetic field ($B$) is equivalent to the CRA mode at a negative magnetic field ($-B$).

As shown in Fig. S10a, the residual cavity mode corresponding to the CRI circular polarization component (blue peak) exhibits a small red-shift with increasing magnetic field. After folding that branch to the negative magnetic field region, it falls nicely on the CRA-UP branch, as presented in Fig. 3b and c. As explained in the main text, this red-shift is enabled by the unusually strong coupling in our system for the detuning $\Delta > \omega_0$. From an alternative viewpoint, the residual cavity mode is actually the upper polariton branch of CRI (CRI-UP), and the red-shift is a result of anti-crossing between CR and the THz cavity mode at $-1$ Tesla, as shown in Fig. S10c. In our experiments, a linearly polarized THz probe beam was used, which resulted in a 50%-50% superposition of CRA and CRI features. Experimental polariton peak positions match well with the classical electromagnetic field simulation, as presented in Fig. S10d.

Figure S10: a, Power transmittance spectra at specific magnetic fields. Lorentzian fits are shown as solid lines. The residual cavity mode (blue peak) exhibits a red shift with increasing magnetic field. b–d, Classical electromagnetic field simulation of THz transmission and experimental polariton peak positions for b, CR active (CRA) circular polarization, c, CR inactive (CRI) circular polarization, and d, linear polarization. Red (blue) open circles are the polariton peak positions for the CRA (CRI) mode.
References

[1] G. C. DeSalvo, W. F. Tseng, and J. Comas, “Etch Rates and Selectivities of Citric Acid/Hydrogen Peroxide on GaAs, Al_{0.3}Ga_{0.7}As, In_{0.2}Ga_{0.8}As, In_{0.53}Ga_{0.47}As, In_{0.52}Al_{0.48}As and InP,” J. Electrochem. Soc. 139, 831 (1992).


