Long-range $p-d$ exchange interaction in a ferromagnet–semiconductor hybrid structure

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A. Origin of the ferromagnet-induced QW PL circular polarization.

Here we discuss more possibilities of FM-induced circularly polarized emission from the QW. We show that these alternative mechanisms cannot explain the experimental data.


We can exclude any potential influence from magnetic circular dichroism (MCD), i.e., the dependence of the absorption coefficient on photon helicity. MCD would induce a PL polarization even if the carriers in the QW would be non-polarized. To check, detection was done at the e-A^0 PL transition (1.494 eV) of the GaAs substrate, and the laser was scanned with alternating helicity across the QW resonances in the range from 1.59 to 1.63 eV with a Faraday-field of \( B_f = \pm 70 \) mT applied. Only negligible changes of the PL intensity <0.3% were found. Therefore MCD is not important and the observed \( A \approx 4\% \) polarization results from the ferromagnet-induced spin polarization of two-dimensional electrons and/or holes.

A2. Role of stray fields in the FM-induced spin polarization.

Optical orientation of electrons occurs under excitation by circularly polarized light. The dependencies of the circular polarization degree \( \rho^c(B) \) on magnetic field in Voigt geometry (the Hanle effect) and in Faraday geometry (stabilization of the optical orientation, recently called the “inverted” Hanle effect [S1]) are determined by the distribution of magnetic stray fields [S2]. The Hanle effect (Fig. S1) is the depolarization of PL in a magnetic field \( B_f \) in Voigt geometry and results from the electron spin precession about \( B_f \). The transversal hole g-factor is close to zero so that the hole depolarization is negligible. The half-width of the depolarization curve \( B^V_{1/2} \approx 15 \) mT in structure 1 for the 7-10 nm range of the spacer thickness. A magnetic field in Faraday geometry increases the circular polarization (“inverted” Hanle effect) with a similar characteristic field \( B^F_{1/2} \approx 30 \) mT. This behavior points towards the presence of stray magnetic fields in the QW region due to the domain structure of the FM [S2] or interface roughness [S1]. (Note that the fringe fields due to hyperfine interaction with the nuclei are about 0.5 mT in CdTe
QWs [S3] and can be safely neglected. The effect of the stray fields is also seen in TRPL data, where they lead to a dephasing of the optical orientation of electrons with a characteristic time $\tau_{\text{deph}} = T_2^* = (1.0 \pm 0.3) \text{ ns}$, see the lower panel of Fig. 3d. For the case of an isotropic distribution of the random fields there is a relation [S4] between $B_{1/2}^V$, $B_{1/2}^F$, and $T_2^*$: 

$$B_{1/2}^F = B_{1/2}^V = 2\sqrt{3} \frac{\hbar}{\mu_B g_e T_2^*}.$$ 

Taking into account $|g_e| = 1.2$ one obtains $B_{1/2}^{V,F} = 22 \text{ mT}$, which is between the $B_{1/2}^V = 15 \text{ mT}$ and $B_{1/2}^F = 30 \text{ mT}$ observed experimentally. The difference due to $B_{1/2}^V \neq B_{1/2}^F$ indicates an anisotropic distribution of the FM random fields. Small local fields in the ten mT range can affect the electron spin precession kinetics. However, the polarization of electrons (and of holes whose longitudinal g-factor has a similar value) due to the thermal distribution between the spin sublevels at $T = 2 \text{ K}$ cannot exceed 0.5% at this field strength, i.e. is too small to explain the experiment.

**A3. Role of resonant tunneling through deep centers in the (Cd,Mg)Te spacer and diffusion of magnetic atoms into the QW region or of cluster formation**

Another possible reason of the FM-induced spin polarization is related to resonant transfer of spin coupling through overlapping deep paramagnetic centers in the (Cd,Mg)Te barrier. Assuming a localization radius of such a center of about $a_i \sim 1 \text{ nm}$ one can estimate the concentration required to make such a coupling efficient to be $N_i \sim 1/a_i^3 \sim 10^{21} \text{ cm}^{-3}$. Such a large concentration of paramagnetic centers was never reported for (Cd,Mg)Te. Moreover, its presence should significantly influence the Larmor precession frequency of the CdTe QW electrons, which was not observed so far. Therefore we consider this possibility as highly unlikely.

Diffusion of magnetic atoms into the QW region could enhance the effective Lande g-factors of spins, so that $g_{\text{eff}} \gg 1$, due to exchange interaction similar to the behavior in diluted magnetic semiconductors. A further possibility for the small saturation field $B_{\text{sat}} \sim 20 \text{ mT}$ of $\rho_e^x(B_F)$ (see Fig. S2) could be superparamagnetic Co cluster formation. For clarification, the polarization dependencies $\rho_e^x(B_F)$ were measured at different temperatures in the range from 2 up to 30 K.
Figure S2 shows that all curves coincide when the measured amplitudes are scaled accordingly. Therefore the polarization originates from a ferromagnet, and not from paramagnetic clusters or Co impurities. In the latter cases, the saturation of \( \rho_c^p(B) \) would depend on the ratio 
\[ M_c B / k_B T \]  
(\( k_B \) is the Boltzmann constant, and \( M_c \) is the magnetic moment of the cluster) or on \( \mu_B B_{\text{eff}} / k_B T \) for Co impurities. With increasing temperature polarization saturation should occur then in large fields, \( B_{\text{sat}}(T) \sim k_B T \). However, the saturation of \( \rho_c^p(B_c) \) takes place at \( B_{\text{sat}} = 20 \) mT over the entire temperature range in Fig. S2. This also means that the ferromagnetism of the Co layer is not sensitive to temperature up to 30 K (the origin of the ferromagnetic behavior will be discussed below). The decrease of polarization with increasing temperature (Fig. 4a) is related to the decreasing sensitivity of the “QW detector” to ferromagnetism.

**B. Origin of the ferromagnet responsible for the spin polarization of the QW holes.**

The magneto-optical Kerr effect is often used to measure the magnetic hysteresis loop of thin magnetic films. In our case, the Kerr data show that the magnetization of the Co film is oriented in-plane due to demagnetization with the saturation field \( B_F \) being about 1.5 T (see Fig. S3a), which is a typical value for \( 4\pi M_s \) in Co. The magnetization by an in-plane magnetic field \( B_y \) perpendicular and parallel to the spacer gradient direction show magnetic hysteresis loops with a coercivity \( B_c = 120 \) mT (see Fig. S3b). No changes within the 20 mT accuracy can be seen in Figs. S3a and S3b. These data are in contrast with the proximity effect: The PL polarization degree \( \rho_c^p \) in perpendicular magnetic field \( B_F \) saturates at \( B_F = 20 \) mT (Fig. 4b of the main text and Fig. S1) and is weakly sensitive to an in-plane magnetic field with \( B_y < 100 \) mT.

We conclude that the Kerr technique and the PL polarization technique detect ferromagnets with different properties – the easy-plane for the former and the perpendicular anisotropy for the latter method. The FM detected by the Kerr effect can be ascribed to the Co
film itself, whereas the other FM could be related to interfacial ferromagnetism with properties differing substantially from the Co film. The interfacial FM layer is expected to be thinner than the Co film thickness, so that it is hard to be detected by the Kerr technique. In contrast, the FM-QW exchange coupling may be sufficient to induce spin polarization of nearby QW charge carriers. Interfacial ferromagnetism was discovered in a Ni/GaAs hybrid [S2] and was discussed later in a number of papers [S5, S6]. It may (and really does) differ from the magnetism of the materials before bringing them into contact. The ferromagnetism of the interface can be determined by the intermixing of FM and SC atoms [S2] and/or spin-spin interactions between FM and magnetic atoms in SC (e.g. Mn in GaMnAs, [S5,S6]) even for an ideal boundary. In our case the intermixing of Co and non-magnetic CdMgTe looks more likely. At this stage, the origin of the interfacial FM in the Co/(Cd,Mg)Te/CdTe hybrids needs further investigation in future. Currently we can exclude Co-related oxides as origin because the structures 2 and 3 were grown without exposure to air before FM sputtering. Interestingly, the FM proximity effect decreases with increasing Co thickness. Figure B3 shows the time-integrated amplitude $A=\rho^2(40\text{m}\Omega)$ versus the Co thickness $d_{\text{Co}}$ for a fixed spacer thickness $d_s=10\text{nm}$ at $T=2\text{K}$. The proximity effect decreases not only for small Co thicknesses $d_{\text{Co}}<4\text{nm}$ (which is natural) but also for $d_{\text{Co}} > 4\text{nm}$. The latter may be related to reorientation of the easy axis or change of the interfacial ferromagnetism, when the Co film becomes thick enough.

**C. Hole spin – acoustic phonon coupling in the semiconductor valence band**

The interaction of a valence band hole spin $\mathbf{J}$ with acoustic phonons through the shear strain is described by the Hamiltonian [S7]

$$H_{\text{ph-h}} = \frac{2d}{\sqrt{3}} \left( [J_z, J_x] e_{xz} + [J_z, J_y] e_{yz} + [J_y, J_x] e_{yx} \right)$$

where $[J_z, J_x] = (J_z J_x + J_x J_z)/2$, and the constant $d$ has a typical value of about 10 eV. We assume for simplicity that the Hamiltonian has no dependence on $x$, $y$, i.e. the phonon
momentum $q || n || z$. Therefore, the components of the strain tensor $\varepsilon_{xx} = \frac{1}{2} \frac{\partial R_x}{\partial x}$, $\varepsilon_{yy} = \frac{1}{2} \frac{\partial R_y}{\partial y}$, $\varepsilon_{zz} = 0$. The $\alpha = x, y$ components of the displacement vector are given by

$$R_{\alpha} = \sum_q \sqrt{\frac{\hbar}{2 \rho \omega_q}} \left( \xi_{\alpha} b_q e^{iqz} + \xi_{\alpha}^* b_q^* e^{-iqz} \right),$$

where $b_q^+(b_q)$ are the creation (destruction) operators of a phonon with momentum $q$ and frequency $\omega_q$, propagating along the $z$ axis, $\xi_{\alpha}(\xi_{\alpha}^*)$ is the complex $\alpha$-component of the polarization vector $\xi$, $\rho$ is the mass density of the crystal (volume of the sample $V=1$). The Hamiltonian takes a convenient form after transformation to the ladder operators $J_z = J_x + iJ_y$ and the circularly polarized basis

$$R_{x} = R_x \pm iR_y = \sum_q \sqrt{\frac{\hbar}{2 \rho \omega_q}} \left[ (\xi_x \pm i\xi_y) b_q e^{iqz} + (\xi_x \pm i\xi_y) b_q^* e^{-iqz} \right].$$

$$H_{ph-h} = \frac{d}{2\sqrt{3}} \left( [J_z,J_x] \frac{\partial R_x}{\partial z} + [J_z,J_y] \frac{\partial R_y}{\partial z} \right) \quad (C1)$$

The first term in Eq. (C1) couples the ground state $|+3/2,N\rangle$ of the heavy hole in the QW in presence of $N$ phonons to the excited state $|+1/2,N-1\rangle$ of the light hole in presence of $N-1$ phonons with a probability amplitude $\langle +1/2,N-1 | H_{ph-h} | +3/2,N \rangle$ via a $\sigma^-_{phon}$ circularly polarized phonon (coefficients $\xi_x = 1/\sqrt{2}, \xi_y = -i/\sqrt{2}$), satisfying angular momentum conservation. Similarly, the second term in Eq. (C1) couples the ground state $|-3/2,N\rangle$ of the QW heavy hole in presence of $N$ phonons and the excited light hole state $|-1/2,N-1\rangle$ in presence of $N-1$ phonons with the probability amplitude $\langle -1/2,N-1 | H_{ph-h} | -3/2,N \rangle$ via a $\sigma^+_{phon}$ phonon. As the number of phonons with opposite circular polarizations is different especially near the magnon-phonon resonance, the probability amplitudes for the two couplings $+3/2 \leftrightarrow +1/2$ and $-3/2 \leftrightarrow -1/2$ are different. Assuming that near the resonance only the $\sigma^-_{phon}$ phonon mode survives, we obtain a shift of the $+3/2$ heavy-hole state due to the “phonon ac Stark effect” (the shift of the $-3/2$ state is absent in this case)
\[ \Delta E_{3/2} = \sum_q \left\{ \frac{\langle +1/2, N_q - 1 | H_{\text{ph-h}} | +3/2, N_q \rangle^2}{\hbar \omega_q - \Delta_{1h}} \right\} \]  
(C2)

where the sum is limited to wave vectors \( q \), for which the dispersion relation of the transverse acoustic phonons is close to the magnon-phonon resonances, \( E_{mp} = \hbar \omega_1, \hbar \omega_2 \). It follows from Eq.(C2) that the shift is maximal when the energy of the magnon-phonon resonance \( \hbar \omega_1, \hbar \omega_2 < 1 \text{meV} \) is close to \( \Delta_{1h} \). For the acceptor the energy splitting \( \Delta_{1h} \leq 1 \text{meV} \) is indeed close to the phonon resonance, in contrast to the \( \sim 10 \text{meV} \) energy splitting between the free hole subbands. Therefore, the spin splitting of the \( \pm 3/2 \) acceptor levels is expected to be larger than that of the QW exciton, in accordance with the stronger polarization of the e-A\(^0\) line. Next we estimate the phonon-induced splitting of the acceptor states.

Using Eq. (C2) we estimate the shift of the \( +3/2 \) level of the hole bound to a neutral acceptor and show that it agrees with the splitting value \( \Delta E_{\text{ex}} = 50 \mu \text{eV} \) and hole polarization 5% extracted from our experiment. We assume that the phonons are \( \sigma^{-}_{\text{phon}} \) circularly polarized in the energy interval \( \delta E \) near the crossing point \( E_{mp} \) of the magnon-phonon resonance [19]. The perturbation due to the spin-phonon Hamiltonian looks similar to Eq.(C1), where the spin \( J \) stands now for the total angular momentum (orbital and spin) of a hole bound to an acceptor with renormalized deformation constant \( d \) [S8]. The acceptor levels in the quantum well are split into two pairs corresponding to the projection of angular momentum \( \pm 3/2 \) and \( \pm 1/2 \) onto the growth direction – the z-axis, with the \( \pm 3/2 \) states being the ground states of the acceptor [23]. First, we estimate the matrix element \( \langle +1/2, N \rangle \hbar \omega_{\text{ph-h}} | +3/2, N \rangle \). Due to the selection rules for \( \sigma^{-}_{\text{phon}} \) polarization only the first term in Eq.(C1) contributes to the level shift. The shear strain

\[ \frac{\partial R}{\partial z} = \sum_q (iq) \sqrt{\frac{\hbar}{\rho \omega_q}} b_q \]  

is practically spatially uniform because \( qa_B << 1 \) (\( a_B \) is the acceptor Bohr radius). Hence,
\[
\langle \frac{1}{2}, N_q - 1 \mid H_{ph-h} \mid \frac{3}{2}, N_q \rangle = \frac{d}{2} \sqrt{3} \langle \frac{1}{2}, [J_z, J] \mid \frac{3}{2} \rangle \langle N_q - 1 \mid \partial R_{z} \mid N_q \rangle = i \frac{d}{2} q \sqrt{\frac{hN_q}{\rho\omega_q}}
\]

With this matrix element we obtain
\[
\Delta E_{3/2} = \frac{d^2\hbar}{4\rho} \sum_q q^2 \frac{N_q/\omega_q}{h\omega_q - \Delta_{th}}
\]

(C3)

The summation over \( q \) is limited to wave vectors, for which the energy of the transverse acoustic phonons is close to the magnon-phonon resonance. Recall that the Eqs. (C1,C2,C3) were written for the case when \( \vec{q} \parallel \vec{m} \). Generally the phonon momentum \( \vec{q} \) is tilted from the \( \vec{m} \) direction by the angle \( \Theta \neq 0 \). The dependence of the ellipticity of the eigenvectors of the magneto-elastic eigenmodes on the angle \( \Theta \) is rather complicated [18]. Symmetry arguments suggest that transverse phonons with \( \vec{q} \perp \vec{m} \) (\( \Theta = \pi / 2 \)) are not elliptically polarized. Only the z-component \( q_z \) of the momentum \( \vec{q} \) transfers angular momentum along \( \vec{m} \). We take this approximately into account by expanding the summation in Eq.(C3) over all \( \vec{q} \) directions and dividing by a factor 3 because only one (z-direction) of the three possible directions is relevant
\[
\Delta E_{3/2} \approx \frac{d^2\hbar}{12\rho} \sum_q q^2 \frac{N_q/\omega_q}{h\omega_q - \Delta_{th}}
\]

(C4)

where the phonon energy \( h\omega_q = h\sigma_q \) is within the range \( \Delta \) around the crossing point \( E = E_{mp} \).

Converting the summation into an integration one gets
\[
\Delta E_{3/2} \approx \frac{d^2\hbar^2}{12\rho} \int g(E) \frac{N(E)}{E-E_{mp}} \frac{(E/h\sigma_q)^2}{E-\Delta_{th}} dE
\]

(C5)

where the phonon density of states \( g(E) = \frac{E^2}{2\pi^2(s\hbar)^3} \), \( s \) is the velocity of transverse acoustical phonons (TA), \( E = h\sigma_q \), and the Planck distribution \( N(E) = \frac{1}{\exp\left(\frac{E}{k_BT}\right) - 1} \). Usually the region
where elliptically polarized phonons are present is small, i.e. $\Delta E \ll E_{mp}$. Hence one can perform the integration in Eq.(C5) by fixing the integrand at $E = E_{mp}$

$$\Delta E_{3/2} \approx \frac{d^2}{12\rho s_i^2} g(E_{mp}) N(E_{mp}) \frac{E_{mp}}{E_{mp} - \Delta_{th}} \Delta E$$  \hfill (C6)

There are two crossing points with $\omega_1 \sim 10^{10} \text{s}^{-1}, \omega_2 \sim 10^{12} \text{s}^{-1}$ [19] corresponding to $\hbar \omega_1 \sim 0.01 \text{meV}$, $\hbar \omega_2 \sim 1 \text{meV}$, respectively. The latter point gives a larger contribution to the $p$-$d$ exchange because the density of phonon states is larger. Another reason is the magnitude being close to the $\Delta_{th} \approx 1 \text{meV}$ [23] splitting of the acceptor states $3/2$ and $1/2$. The ac Stark shift $\Delta E_{3/2}$ of the $-3/2$ state is absent for $\sigma_{phon}^{-}$ phonons. Hence the shift $\Delta E_{3/2}$ gives the $p$-$d$ splitting $\Delta E_{ex}$ of the $\pm 3/2$ states. We come to the following formula to estimate the $p$-$d$ exchange coupling strength for $E_{mp} = \hbar \omega_2$

$$\Delta E_{ex} = \Delta E_{3/2} \approx \frac{d^2}{12\rho s_i^2} \omega_2^2 \frac{1}{2\pi s_i^3} \exp \left(-\frac{\hbar \omega_2}{k_B T}\right) - 1 \frac{\Delta E}{\hbar \omega_2 - \Delta_{th}}$$  \hfill (C7)

Inserting into Eq.(C7) the values $\omega_2 = 1.5 \cdot 10^{12} \text{s}^{-1}$, $\Delta E / \hbar \omega_2 = 0.05$ (see below) for the ferromagnet, $\rho = 5.87$ $\text{g/cm}^3$, $s_i = 1.79 \cdot 10^5$ $\text{cm/s}$ for CdTe [S9], the deformation constant $d = 5$ $\text{eV}$ [S8], the detuning $|\Delta_{th} - \hbar \omega_2| = 0.1 \text{meV}$ and the temperature $T = 6 \text{K}$ one gets $\Delta E_{ex} \approx 50 \mu\text{eV}$ and a polarization $P = \frac{\Delta E_{ex}}{2k_B T} \approx 5\%$. These numbers are in good agreement with the experiment in Fig.4a at low temperature (the data point $A = 4.2\%$ at nominal $T = 2 \text{K}$ seems also in accord with the estimate if we keep in mind the possible heating of the sample by a few K due to the illumination). Equation (C7) predicts an increase of $\Delta E_{ex} \sim T$ at higher temperature $T > 5 \text{K}$, because the number of phonons $N_q \approx \frac{k_B T}{\hbar \omega_2}$. The polarization of holes should not depend on $T$. In contrast to it, the polarization decreases as Fig.4a shows. This means that our simple model overestimates the polarization value at high $T$. Another process, however, may become important
and decrease the FM-SC coupling strength at higher temperature: although shear waves effectively cross the FM/SC boundary [20] the degree of the phonon circular polarization may decrease (recall that the phonons in bulk semiconductors are linearly polarized).

The width \( \delta E \) of the magnon-phonon resonance can be estimated using Eq.(42) in Ref.[19]:

\[
(\omega - \omega') (\omega^2 - s_i^2 q^2) = \frac{\gamma M q^2}{\rho} \left( \frac{b_2}{M} \right)^2.
\]

The magnon-phonon resonance takes place near the crossing point of the spin wave \( \omega'(q) \) and phonon branches \( \omega'(q) = s_i q \equiv \omega_z \). Considering the right-hand term as a small perturbation and using \( \omega'^2 - \omega_z^2 \approx 2 \omega_z (\omega - \omega_z) \) one obtains

\[
\delta E = 2|\omega - \omega_z| = \hbar \frac{2 \gamma M \omega_z b_2}{\rho S_f^2 M},
\]

where the gyromagnetic ratio \( \gamma \approx 2 \cdot 10^7 \text{ s}^{-1}/\text{Oe} \), the magnetoelastic constant \( b_2 = 10^8 \text{ erg/cm}^3 \), and the magnetization \( M = 500 \text{ Oe} \) taken from the values for typical ferromagnets (the parameters characterizing the interfacial magnetism are unknown so far). Using the FM density \( \rho_f = \rho = 5.7 \text{ g/cm}^3 \) and transverse sound velocity \( S_f = s_i = 1.79 \cdot 10^5 \text{ cm/s} \) one obtains \( \delta E = 0.05 \text{ meV} \) which is 5% of the energy \( \hbar \omega_z = 1 \text{ meV} \). We have used this \( \delta E \) value in Eq.(C6).

### D. Coupling of hole spin with optical phonons

One cannot exclude the influence of elliptically polarized transversal (TO) optical phonons (corresponding to the zero-point motion in the low temperature case). Coupling with circularly polarized optical phonons is similar to a spin-dependent polaronic shift. Circularly polarized \( \sigma_{\text{phon}}^+ \) and \( \sigma_{\text{phon}}^- \) optical phonons with different frequencies \( \omega_0^+ \) and \( \omega_0^- \) are observed in magnetic field with their energies being split by \( \sim 10 \text{ cm}^{-1} \) [17]. To illustrate this point we consider for simplicity a crystal with two atoms per unit cell. Uniform strain is characterized by the relative displacement \( \mathbf{r} = \mathbf{R}_1 - \mathbf{R}_2 \) of the sublattices 1 and 2. Unlike the acoustic phonon case, optical oscillations do not shift the center of mass of the unit cell. Hence the spin-phonon
Hamiltonian includes the displacement itself, without derivatives (being typically much smaller) with respect to the coordinates

\[ H_{\text{opt-h}} = Q_{\text{opt-h}}\left[r_x J_z J_y + J_z J_x + J_y J_x \right] \]

(D1)

The third rank tensor \( \hat{Q} \) has only one independent component \( Q = Q_{\text{opt-h}} = Q_{\text{opt-h}} = Q_{\text{opt-h}} \) (of dimensionality eV/A) in CdTe-type bulk semiconductors. The order of magnitude of the parameter \( Q \sim \epsilon^2/a_0^4 \) is a few eV/A \( (a_0 \) is the linear size of the unit cell). Optical phonon modes propagating along \( \mathbf{z} || \mathbf{m} \) (the structure growth axis) transfer angular momentum parallel to \( \mathbf{m} \).

The angular momentum is related to the \( x \) and \( y \) components of the displacement vector \( \mathbf{r} \). For this reason we omit the last term in Eq. (D1). Similar to (C1) it is convenient to transform the Hamiltonian (D1) to the operators \( J_x = J_x \pm iJ_y \) and the circularly polarized basis \( r_x = r_x \pm ir_y \) after which one obtains

\[ H_{\text{opt-h}} = \frac{Q}{2i}\left(r_x [J_z, J_x] - r_x [J_z, J_x] \right) \]

(D2)

We consider the low-temperature case \( \Delta_{ph}, \hbar \omega_0 >> k_B T \), when only the 3/2 heavy hole states are populated and there are no optical phonons. The shift of the energy level of the +3/2 hole follows from second-order perturbation theory

\[ E_{+3/2} = -\sum_q \frac{\left( + \frac{1}{2}, \frac{1}{2}, q \left| H_{\text{opt-h}} + 3/2, 0, q \right| \right)^2}{\hbar \omega_0^+ + \Delta_{ph}} \]

(D3)

where the matrix element in Eq. (D3) couples the ground +3/2 state without phonons with the excited +1/2 hole state with one phonon \( \hbar \omega_0 \) having momentum \( \mathbf{q} \) along \( \mathbf{z} \) and \( \sigma_{\text{phon}}^+ \) polarization (the second term in Eq.(D2) is responsible for this coupling). The upper index in the phonon frequency \( \omega_0^+ \) takes explicitly into account the broken time-reversal symmetry in the FM/SC hybrid, so that the phonon energies \( \hbar \omega_0^+ \) and \( \hbar \omega_0 \) are different for \( \sigma_{\text{phon}}^+ \) and \( \sigma_{\text{phon}}^- \) polarizations. Similar to Eq.(D3) one can write the shift of the ground -3/2 state as...
where the matrix element in Eq. (D3) couples the ground $-3/2$ state without phonons with the excited $-1/2$ hole state with one phonon $1_q$ having momentum $q$ along $z$ and $\sigma^-$ polarization (the first term in Eq. (D2) is responsible for this coupling). Using Eqs. (D3, D4) we obtain for a small splitting $|\hbar \omega^+_0 - \hbar \omega^-_0| << \hbar \omega_0$ of the TO modes

$$\Delta E^{\text{opt}}_{\text{ex}} = E_{+3/2} - E_{-3/2} = \beta (\hbar \omega^+_0 - \hbar \omega^-_0)$$

(D5)

where the coefficient $\beta \sim \left| \langle 1 | H_{\text{opt}} - h \rangle | 0 \rangle \right|^2 / (\hbar \omega_0)^2 < 1$. The difference $|\hbar \omega^+_0 - \hbar \omega^-_0|$ and the parameter $\beta$ are not known with high precision for the studied hybrid. Taking as a reference point $|\hbar \omega^+_0 - \hbar \omega^-_0| = 20 \text{ cm}^{-1}$ [17] and our estimation $\Delta E_{\text{ex}} = 50 \mu\text{eV}$ one should have $\beta \sim 0.01$. 

\[
E_{-3/2} = -\sum_q \left| \langle -1/2, 1_q | H_{\text{opt}} - h \rangle | -3/2, 0_q \rangle \right|^2 / \hbar \omega^+_0 + \Delta_{1h},
\]

(D4)
Figure S1. Magnetic field dependence of circular polarization. Hanle effect (the circles) in Voigt geometry, fitted with a Lorentzian with $B^{\text{V}}_{1/2} \approx 15$ mT. “Inverted” Hanle effect (the squares), also fitted by a Lorentzian with $B^{\text{F}}_{1/2} \approx 30$ mT; $T = 10$ K, data recorded on structure 1.
**Figure S2. Effective p-d exchange interaction between FM and QW heavy holes.**

Dependence $\rho^p_c(B_F)$ on magnetic field at different temperatures for a spacer thickness of 7.5 nm; all curves are recalibrating by rescaling the vertical scale of the data to the same average circular polarization value.
Figure S3. (a) The polar Kerr effect reveals the out-of plane magnetization component in Faraday field $B_F$. (b) The longitudinal Kerr effect detects the in plane magnetization component for two different $B_F$ field orientations, parallel and perpendicular to the spacer thickness gradient. (c) Time-integrated amplitude $A=\rho_n(B_F=40\text{mT})$ versus Co thickness $d_{Co}$ for $d_s=10\text{ nm}$ at $T=2\text{ K}$. 
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