Generation and detection of pure valley current by electrically induced Berry curvature in bilayer graphene

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I. Measurement error due to the current leakage

A finite input impedance of a voltage amplifier which was used in the measurement circuit causes artifacts due to the current leakage which we should distinguish from the real signal. In Fig. S1a we show a device in which such a current leakage effect was clearly observed. The nonlocal resistance $R_{\text{NL}}$ was measured by injecting current from terminal 8 to 5 and measuring voltage difference between terminal 4 and 3. Local resistivity $\rho$ was measured simultaneously by detecting the voltage drop between terminal 7 and 6. The nonlocal voltage was measured after being amplified with a commercial differential voltage amplifier (SR560) with a 100 MΩ input impedance. Fig. S1b describes a simple circuit model for the device and measurement setup. There is a finite current flowing into the voltage amplifier because of the finite input impedance between the inputs and the ground. If the resistances $R_A$ and $R_B$ are not equal, a finite voltage difference between the two inputs emerges and is detected as a nonlocal voltage. At the charge neutrality point, the dominant source of $R_A$ and $R_B$ is the device resistance under the top gate, because the other device area is highly doped by the back gate voltage. Because of the asymmetry in the device geometry (Fig. S1a), $R_A$ and $R_B$ are not equivalent and therefore a finite nonlocal voltage reflecting the nonlocal resistance of

$$R_{\text{NL}} = \left( \frac{R_m}{R_m + R_B} - \frac{R_m}{R_m + R_A} \right) R_C \sim \frac{R_A - R_B}{R_m} R_C, \quad (S1)$$

emerges. There is a characteristic scaling relation between $R_{\text{NL}}$ and $\rho$, i.e. $R_{\text{NL}} \propto \rho^2$ in this situation because $R_A$, $R_B$, and $R_C$ are almost proportional to the local resistivity $\rho$ of the device.

Fig. S2 shows the experimental observation of the square scaling relation between $R_{\text{NL}}$ and $\rho$ in the device shown in Fig. S1a. The measurement was performed with a DC setup. As described above, the characteristic scaling relation $R_{\text{NL}} \propto \rho^2$ due to the current leakage was observed. When $\rho \sim 100 \text{ kΩ}$, i.e. $R_C \sim 400 \text{ kΩ}$ and $R_A - R_B \sim$ a few 100 kΩ from the aspect ratio, $R_{\text{NL}} \sim 10^3 \text{ Ω}$ is obtained from equation (S1), which is also consistent with the experimental result. On the other hand, neither cubic scaling nor saturation feature of $R_{\text{NL}}$ was observed. This is probably due to the reflection of valley current at the interface between the top-ungated doped region and the top-gated charge neutral region.

The current leakage effect described above can be present in the nonlocal resistance measurement in the main text as well. Here the impedance $Z_{\text{in}}'$, which causes the leakage
current, includes not only the input impedances of the voltage preamplifiers but also parasitic capacitances of the measurement system. For the measurement of $R_{45,67}$, $Z^i_{in}$ are $Z^1_{in} = 1/(1/R_{in} + i\omega C_p)$ and $Z^2_{in} = 1/(3/R_{in} + i\omega C_p)$, where $R_{in} = 100 \text{ M}\Omega$ is the input impedance of the voltage preamplifier and $C_p = 7.2 \text{ nF}$ is the parasitic capacitance of the measurement system. Note the dominant source of $C_p$ is the commercial low pass filter. The nonlocal impedance $Z^{\text{leak}}_{45,67}$ due to the current leakage to the ground is given by

$$Z^{\text{leak}}_{45,67} = \left( \frac{Z^1_{in}}{Z^1_{in} + R_A} - \frac{Z^2_{in}}{Z^2_{in} + R_B} \right) R_D \approx \left( \frac{R_B}{Z^2_{in}} - \frac{R_A}{Z^1_{in}} \right) R_D$$

$$= \frac{3R_B - R_A}{R_{in}} R_D + i\omega C_p (R_B - R_A) R_D. \quad (S2)$$

What we measure is the nonlocal impedance $Z^{\text{meas}}_{45,67}$ defined as the sum of the nonlocal resistance induced by the valley current and the above described $Z^{\text{leak}}_{45,67}$. The imaginary part of the measured nonlocal impedance is not affected by valley current mediated nonlocal resistance, but only comes from the current leakage effect due to the parasitic capacitance. So the following equation holds:

$$\text{Im} Z^{\text{meas}}_{45,67} = \text{Im} Z^{\text{leak}}_{45,67} = \omega C_p (R_B - R_A) R_D. \quad (S3)$$

We also measure the two-terminal resistance between the terminals 6 and 7,

$$R_{6,7} = R_C + R_D. \quad (S4)$$

Similarly, for the measurement of $R_{67,45}$,

$$Z^{\text{leak}}_{67,45} \approx \frac{3R_B - R_C}{R_{in}} R_B + i\omega C_p (R_D - R_C) R_B, \quad (S5)$$

and

$$\text{Im} Z^{\text{meas}}_{67,45} = \text{Im} Z^{\text{leak}}_{67,45} = \omega C_p (R_D - R_C) R_B. \quad (S6)$$

We also measure the two-terminal resistance,

$$R_{4,5} = R_A + R_B. \quad (S7)$$

By solving the equations of (S3), (S4), (S6), and (S7), we obtain the following relations for
\( R_A, R_B, R_C, \) and \( R_D \):

\[
R_A = \frac{b}{4} \left\{ 3 - \frac{2(c-d)}{\omega C_p ab} - \sqrt{1 + \left( \frac{2(c-d)}{\omega C_p ab} \right)^2 + \frac{8d}{\omega C_p ab}} \right\},
\]

\[
R_B = \frac{b}{4} \left\{ 1 + \frac{2(c-d)}{\omega C_p ab} + \sqrt{1 + \left( \frac{2(c-d)}{\omega C_p ab} \right)^2 + \frac{8d}{\omega C_p ab}} \right\},
\]

\[
R_C = \frac{a}{4} \left\{ 3 + \frac{2(c-d)}{\omega C_p ab} - \sqrt{1 - \left( \frac{2(c-d)}{\omega C_p ab} \right)^2 + \frac{8c}{\omega C_p ab}} \right\},
\]

\[
R_D = \frac{a}{4} \left\{ 1 - \frac{2(c-d)}{\omega C_p ab} + \sqrt{1 - \left( \frac{2(c-d)}{\omega C_p ab} \right)^2 + \frac{8c}{\omega C_p ab}} \right\}. \tag{S8}
\]

where \( a, b, c, \) and \( d \) denote \( R_{6,7}, R_{4,5}, \text{Im} Z_{45,67}^{\text{meas}}, \) and \( \text{Im} Z_{67,45}^{\text{meas}}, \) respectively. Therefore the values of \( a, b, c, \) and \( d \) are all experimentally derived. By using them, we calculate the real part of the nonlocal impedance due to the current leakage effect using the following relations,

\[
\text{Re} Z_{45,67}^{\text{leak}} = \frac{3R_m - R_A}{R_m} R_D, \tag{S9}
\]

and \( \text{Re} Z_{67,45}^{\text{leak}} = \frac{3R_m - R_C}{R_m} R_B. \tag{S10} \)

By subtracting this real part from the real part of the measured nonlocal impedance, we obtain the nonlocal resistance due to the valley Hall effect as the corrected values of

\[
R_{45,67}^{\text{corrected}} = \text{Re} Z_{45,67}^{\text{meas}} - \text{Re} Z_{45,67}^{\text{leak}}, \tag{S11}
\]

and \( R_{67,45}^{\text{corrected}} = \text{Re} Z_{67,45}^{\text{meas}} - \text{Re} Z_{67,45}^{\text{leak}}. \tag{S12} \)

The correction is usually very small as shown in Fig. S4 for \( R_{\text{NL}} \) \((= R_{45,67})\) as a typical example. After all we see that the current leakage effect does not alter our conclusion. Therefore in the main text, we only showed the data without correction or real part of the measured nonlocal impedance.
Figure S1 | Asymmetric device and the resistor model which explains the measurement error due to current leakage. a, The AFM image of the asymmetric device before encapsulation with an h-BN flake. The device has a dual gate structure with two h-BN insulating layers. The left and right region in light blue indicate the left top gate (LTG) and the right top gate (RTG), respectively both of which are placed after encapsulation with the h-BN flake. b, The circuit model of the measurement setup. This model describes the measurement error associated with the current leakage effect via the finite input impedance of the voltage preamplifier. The BLG device is represented as the five resistors ($R_A$ to $R_E$) connected with each other. Current $I$ is injected via $R_D$ into the BLG. A differential voltage amplifier, which has the input impedance $R_{in}$ between each input and the ground, is used to amplify the nonlocal voltage. Each arrow in the figure indicates the current flow.
Figure S2 | Scaling relation between $R_{NL}$ and $\rho$ in the asymmetric device. A scaling relation of $R_{NL}$ vs. $\rho$ was obtained at the charge neutrality point at 100 K for various displacement fields in the same way as Fig. 4 in the main text. LTG and RTG voltages (see Fig. S1a) were equivalent and tuned simultaneously. $\rho$ was determined by the 4-terminal resistance $R_{76,85}$. The red (blue) points were obtained for the positive (negative) displacement field.
Figure S3 | Measurement circuit diagram of nonlocal resistance $R_{45,67}$ (a) and $R_{67,45}$ (b) using an AC technique. a, b, To discuss the current leakage effect, total series resistance of the device and the measurement setup is modeled with five resistors of $R_A$ to $R_E$. Each terminal is connected to a commercial low pass filter (BLP-1.9+ from Mini-Circuits) to remove high frequency noise (but not shown for terminal 3 and 8). In a, the AC voltage reduced by a 1/1000 voltage divider is applied with a lock-in amplifier LIA4 to inject an AC current to terminal 6 and 7. A buffer resistor is inserted to suppress possible influences from self-heating in the doped regime. The injected current is amplified by a current preamplifier and measured with LIA4, while the nonlocal voltage between terminal 4 and 5 is detected, using differential voltage amplifier of Vamp1. The voltage between terminal 3 (8) and 5 is detected using Vamp2 (Vamp3) at the same time. $Z_{in}^1$ and $Z_{in}^2$ denote the effective input impedance including not only the input impedance...
of the voltage preamplifiers, but also the parasitic capacitance of the measurement system. After amplification of the voltage via Vamp1, 2, and 3, the voltage difference is measured by the lock-in amplifier LIA1, 2, and 3, respectively. The nonlocal voltage signal is then obtained via LIA1. Both in phase (resistive) and out of phase (capacitive) parts of the signal are monitored here. In the measurement of $R_{67,45}$ in b, the electrical connections to the respective terminals are swapped for those in the measurement of $R_{45,67}$.

![Graph](image_url)

**Figure S4 | Scaling relation of $R_{NL} (= R_{45,67})$ and $\rho$ with and without the current leakage effect correction.** The red (blue) points show the scaling relation between $R_{NL}$ and $\rho$ with (without) the current leakage effect correction which was obtained for the charge neutrality point by modulating the displacement field at 70 K. The blue points are the same as the measurement points in Fig. 4 of the main text.
II. Nonlocal resistance measured by swapping injection and detection terminals

Fig. S5 shows the $R_{\text{NL}}$ along the green axis of Fig. 2b at 70 K obtained for, $R_{\text{NL}} = R_{45,67}$ and $R_{67,45}$ before, and after the injection and detection terminals were swapped, respectively. For almost any gate voltages, the two nonlocal resistances coincide, i.e. $R_{45,67} = R_{67,45}$ as predicted from Onsager reciprocal relations. Slight deviation is probably due to the charge puddle reformation by gate sweeping.

Figure S5 | Nonlocal resistance measured by swapping injection and detection terminals. a-c, The blue (red) curve was obtained by injecting current from terminal 6 to 7 (4 to 5) and detecting voltage between terminal 4 and 5 (6 and 7) (See Fig 1c.) The top and back gates were biased to modify the carrier density along the green axis of Fig. 2b in the main text. The measurement temperature was fixed at 70 K.
III. Channel length dependence of nonlocal resistance

Fig. S6 shows the channel length \( (L) \) dependence of nonlocal resistance. With increasing the distance or \( L \) between the injection and detection terminals, the nonlocal resistance decreases. For \( L=8 \mu m \), no clear nonlocal resistance signal was observed in our measurement resolution. The reduced nonlocal resistance with increasing \( L \) is most likely explained by inter-valley scattering. The dominant source of the inter-valley scattering in graphene has been considered to be edge scattering\(^{2,3} \). In this case the scattering length is in the order of the channel width \( W \) (=1 \( \mu m \)). Then the observed nonlocal resistance peak may be more significantly reduced for the larger \( L \).

**Figure S6 | Channel length dependence of nonlocal resistance.** The DC measurement of nonlocal resistance was performed at 73 K for the same BLG device but using different channel length \( (L) \) Hall bars (see Fig. 1c in the main text). While sweeping \( V_{TG}, V_{BG} \) was fixed at -50 V. Blue, green, and red curves are the nonlocal resistances \( R_{45,67}, R_{23,45}, \) and \( R_{23,67} \) obtained for \( L=3.5 \mu m, 4.5 \mu m, \) and \( 8 \mu m \), respectively. For \( L=8 \mu m \), the signal intensity was smaller than the measurement resolution.
IV. Derivation of activation energy of local resistivity

The temperature dependence of the maximum local resistivity for each fixed displacement field $D$ was fit to the double exponential function of equation (4) in the main text. When the data points are plotted with respect to the inverse temperature, the intervals of the data points are not uniform as shown in Fig. 3a. To extract the activation energy $E_{11}^L$, the data points were not weighted according to the density of the data points. This is because the data points in the high temperature regime are dense and the fitting without the weighting allows more correct estimation of $E_{11}^L$. The extracted $E_{11}^L$ for each $D$ is shown in Fig. S7. This $E_{11}^L$ corresponds to $\Delta$ (half of the band gap) and linearly increases with $D$, indicating the band gap size increased up to $2\Delta \sim 80$ meV for the large $D$. On the other hand, $E_{11}^L$ saturates for the region of small $D$. This saturation is attributed to the mobility edge effect$^4$.

Note that to extract the crossover temperature in Fig. 3a, the data points were weighted according to the density of the data points.

![Graph showing displacement field dependence of local resistivity activation energy $E_{11}^L$.](image)

**Figure S7 | Displacement field dependence of local resistivity activation energy $E_{11}^L$.** The activation energy $E_{11}^L$ of the local resistivity is derived from the fitting of the temperature dependence presented in Fig. 3a of the main text. The blue line is a linear fit for the data points.
V. Derivation of activation energy of nonlocal resistance and crossover behavior of nonlocal resistance in displacement field dependence

The temperature dependence of the maximum nonlocal resistance for each fixed \( D \) was fit to the double exponential function of equation (5) in the main text. In the same way as for the local resistivity, the data points were weighted to extract the crossover temperature but not weighted to extract \( E_{1L}^{NL} \). Fig. S8 shows the extracted activation energy \( E_{1L}^{NL} \) for each \( D \). \( E_{1L}^{NL} \) scales linearly with \( D \). However, the extrapolation of the fitting line in blue to \( D = 0 \) shows a small offset energy. The origin of this offset is not yet understood. By applying equations (4) and (5) to the cubic scaling in equation (6): \( R_{NL} \propto \rho^3 \), we expect the relationship between activation energies to be: \( E_{1L}^{NL} = 3E_{1L} \). In fact, the ratio of the slopes of the fitting lines is \( 3.13 \pm 0.36 \) for \( D \gtrsim 0.2 \text{ V/nm} \) meaning: \( \frac{dE_{1L}^{NL}}{dD} \sim 3 \frac{dE_{1L}}{dD} \). However, due to the zero offset of \( E_{1L}^{NL} \), the absolute relation \( E_{1L}^{NL} = 3E_{1L} \) does not hold for the small \( D \). This observation also supports that the observed nonlocal resistance comes from the valley current mediated nonlocal transport.

Note that in ref. 3, similar activation energy between local and nonlocal resistance was found, while our results show discrepancy between them.

The crossover behavior we observed for the \( T \) dependence of \( R_{NL} \) in Fig. 3b of the main text was also observed for the \( D \) dependence of \( R_{NL} \). Fig. S9 shows the \( D \) dependence of \( 1/R_{NL}^{max} \) obtained at 70 K. The data points were well fitted to the function:

\[
\frac{1}{R_{NL}^{max}} = \frac{1}{R_1} \exp\left(-\frac{D}{D_1}\right) + \frac{1}{R_2}. \quad (S13)
\]

In this figure we define the crossover displacement field \( D_c \) at the point where the first term (green line) and the second term (red line) of equation (S13) intersect.

\( D_c \) extracted for each \( T \) in this way is plotted in Fig. S10 together with \( T_c \) extracted for each \( D \) (see Fig. 3c in the main text). The \( D\text{--}1/T_c \) and \( D_c\text{--}1/T \) curves show good correspondence. We assume that there is a crossover phase boundary of nonlocal resistance in the \( D\text{--}1/T \) plane, necessitating the existence of critical points \( D_c(T) \) and \( T_c(D) \). So good correspondence between the \( D\text{--}1/T_c \) curve and the \( D_c\text{--}1/T \) curve, as obtained in Fig. S10, supports the validity of our estimation of \( T_c \) in the main text. However, for a smaller displacement field around 0.2 V/nm, the deviation of the \( D_c\text{--}1/T \) curve from the \( D\text{--}1/T_c \) curve was observed (see Fig. S10 inset). The reason for this deviation is yet to be revealed.
Figure S8 | Displacement field dependence of the nonlocal resistance activation energy $E_{1}^{NL}$. The activation energy $E_{1}^{NL}$ of the nonlocal resistance is derived from the fitting of the temperature dependence presented in Fig. 3b of the main text. The blue line is the linear fit for the data points.
Figure S9 | Fitting example of the displacement field dependence of nonlocal resistance. The inverse of the maximum nonlocal resistance $1/R_{NL}^{\text{max}}$ is obtained at each $D$ when the carrier density is varied at a fixed temperature.
Figure S10 | Relation between nonlocal resistance crossover temperature and crossover displacement field. The blue points show the $1/T_c$ vs. $D$ where $T_c$ was obtained from the $T$ dependence of $1/R_{NL}^{max}$ for each fixed $D$. The red points show the $D_c$ vs. $1/T$ where $D_c$ was obtained from the $D$ dependence of $1/R_{NL}^{max}$ for each fixed $T$. Some red points where $D$ is around 0.2 V/nm are exceeding the range. Inset: All data points are shown in the log scale. Here the deviation of $D_c - 1/T$ curve from the $D - 1/T_c$ curve was observed where $D$ is around 0.2 V/nm.
VI. $R_{\text{NL}} - \rho$ scaling relation obtained near the charge neutrality point

In the main text, a clear scaling relation between $R_{\text{NL}}$ and $\rho$ at the charge neutrality point and $T = 70 \text{ K}$ is shown in Fig. 4. A similar scaling relation was obtained for several gate configurations near (but away from) the charge neutrality point as shown in Fig. S11. Fig. S11a-c show cubic scaling in the smaller displacement field regime.

In Fig. S11d, the scaling starts to deviate from the cubic scaling. An extrinsic mechanism of the valley Hall effect that gives a deviation from what is expected from the intrinsic mechanism model is possible.

**Figure S11 | Scaling relation of $R_{\text{NL}}$ vs. $\rho$ obtained near the charge neutrality point.**

**a-d.** Each data point of $R_{\text{NL}}$ vs. $\rho$ was obtained from Fig. 2a and 2b in the main text by isolating only the $D$ dependence of $R_L$ and $R_{\text{NL}}$ (taking a cut along the red axis) for four different gate configurations, all near the charge neutrality point. Insets: The blue curve shows $R_{\text{NL}}$ data cut along the green axis in Fig. 2b. Only the case for the highest $D$ is shown. The $R_{\text{NL}}$ vs. $\rho$ in each main panel was obtained such that for every $D$, the resistance data was taken at the red broken line.
VII. $R_{NL} - \rho$ scaling relation obtained from temperature dependence

In the main text, the scaling relation between $R_{NL}$ and $\rho$ is obtained from the $D$ dependence at a fixed temperature ($T = 70K$). The cubic scaling holds as far as $\sigma_{xy}^{VH}$ and $l_v$ are independent of parameters such as $D$ and $T$ (see equation (5)). Fig. S12 shows the scaling relation between $R_{NL}$ and $\rho$ obtained from the temperature dependence at the charge neutrality point for two fixed $D$ values. In the low resistivity (high temperature) regime, we find a nonlinear scaling relation which is close to the cubic scaling in green. This result is consistent with the discussion in section V.

![Graphs](https://example.com/graphs.png)

**Figure S12 | Scaling relation of $R_{NL}$ vs. $\rho$ obtained from the temperature dependence at the charge neutrality point.** a and b, Each data point of $R_{NL}$ vs. $\rho$ was obtained from the $T$ dependence of $\rho$ and $R_{NL}$ at the charge neutrality point for fixed $D$. 
VIII. Derivation of nonlocal resistance scaling for the edge dominant transport

To take the nonlocal transport mediated by localized edge states into account, we employed a resistor network model. This kind of resistor network model was previously used to describe the nonlocal transport in an imperfect 2D topological insulator which has a dissipative edge channel and the residual bulk conduction. Here we assume the case where the edge transport is dominant and the lateral transport via the bulk states is negligible because we already discussed the opposite case where the nonlocal resistance is given by van der Pauw formula for the bulk dominant transport in the main text. Fig. S13 shows the resistor network circuit modeled here. By using transmission matrices and taking the limit of $\Delta x \to 0$ (see Fig. S13), we obtain

$$R_{NL} = \frac{\sinh(l_1 / \lambda) \sinh(l_2 / \lambda)}{\sinh((L+l_1+l_2) / \lambda)} \sqrt{2\rho_{bulk} \rho_{edge} W},$$  \hspace{1cm} (S14)

with $\lambda = \sqrt{\rho_{bulk} W / 2 \rho_{edge}}$.

For $\lambda \ll l_1, l_2$, equation (S14) is approximated by

$$R_{NL} = \sqrt{\frac{\rho_{bulk} \rho_{edge} W}{2}} \exp\left(-\frac{L}{\lambda}\right),$$  \hspace{1cm} (S15)

while for $\lambda \gg l_1, l_2$, it is approximated by

$$R_{NL} = \frac{2l_1 l_2}{L + l_1 + l_2} \rho_{edge},$$  \hspace{1cm} (S16)

On the other hand, the local resistance is given by the parallel connection of the upper and lower edge resistances as,

$$R_L = \frac{\rho_{edge} L}{2}.$$  \hspace{1cm} (S17)

In the analysis in the main text, we defined two dimensional resistivity as $\rho = R_L / (L/W)$, and then it is given by

$$\rho = \frac{1}{2} \rho_{edge} W.$$  \hspace{1cm} (S18)

So for $\lambda \ll l_1, l_2$, from equations (S15) and (S18), the relation between $R_{NL}$ and $\rho$ is given by the following formula.

$$R_{NL} = \sqrt{\rho_{bulk} \rho} \exp\left(-\frac{2L}{W} \sqrt{\frac{\rho}{\rho_{bulk}}}ight).$$  \hspace{1cm} (S19)
In equation (S19) \( \rho_{\text{bulk}} \) also depends on \( D \), so that the actual scaling relation of \( R_{\text{NL}} \) vs. \( \rho \) should be more complicated.

On the other hand, when \( \lambda \gg l_1, l_2 \), from equations (S16) and (S18), the scaling relation between \( R_{\text{NL}} \) and \( \rho \) is given by

\[
R_{\text{NL}} = \frac{4l_1 l_2}{(L + l_1 + l_2)W} \rho \propto \rho. \tag{S20}
\]

In either case, the scaling relation obtained here cannot be applied for the observed cubic scaling. Therefore we exclude the possibility of the edge mediated nonlocal transport in the present study.

**Figure S13 | Resistor network circuit model for nonlocal edge transport.** Uniform edge resistivity \( \rho_{\text{edge}} \), and bulk resistivity \( \rho_{\text{bulk}} \) are assumed to model the edge transport. Current \( I \) is injected from the lower side of the network circuit and ejected from the upper side. The nonlocal voltage \( V_{\text{NL}} = V_{\text{NL}}^+ - V_{\text{NL}}^- \) is measured between two points each separated by \( L \) from the current injection and ejection terminal, respectively. The distance from the current injection (voltage probe) terminals to the left (right) boundary of the network circuit is \( l_1, l_2 \) and the circuit width from the current injection to ejection is \( W \). The circuit is divided into sections of length \( \Delta x \), and we finally take the limit of \( \Delta x \to 0 \). The resistor at each section is \( R_1/2 = \rho_{\text{edge}} \Delta x / 2 \) in red, \( R_2 = \rho_{\text{bulk}} W / \Delta x \) in green. In the experimental system, the left and right boundaries are highly doped with the back gate voltage. So the boundary resistance \( R_3 \) in cyan is much smaller than the bulk and the edge resistance, and therefore approximated as \( R_3 = 0 \).
IX. Self-consistent model of valley current mediated nonlocal transport

The nonlocal resistance which results from the combination of the spin Hall effect and the inverse spin Hall effect was previously calculated for a sequential conversion picture\(^7\). In this picture, an injected current induces a transverse spin current and the spin current induces a nonlocal voltage. This is correct when the spin Hall angle \( \alpha_{\text{sh}} = \sigma_{xy}^{\text{SH}} / \sigma_{xx} \) fulfills the condition of \( \alpha_{\text{sh}} \ll 1 \). In ref. 3, this picture was applied to the valley current mediated nonlocal transport and the nonlocal resistance observed for a doped region where \( \alpha_{\text{VH}} = \sigma_{xy}^{\text{VH}} / \sigma_{xx} \leq 1 \) holds was analyzed. In contrast, we discuss the nonlocal transport in the gap. If we only consider the intrinsic valley Hall effect, the valley Hall conductivity in the gap is \( \sigma_{xy}^{\text{VH}} = 4e^2 / h \) as discussed in the main text. So for a large displacement field, the valley Hall angle exceeds one. In such a case, the sequential conversion picture fails. We need to treat the problem in a self-consistent way with a certain boundary condition.

Fig. S14 shows the schematic picture of the nonlocal transport mediated by the valley current which holds for our system. Chemical potential difference \( \delta \mu_v \) between the chemical potentials, \( \mu_k \) and \( \mu_k' \) of two valleys K and K', which can be interpreted as the valley voltage, is defined as \( \delta \mu_v = \mu_k - \mu_k' \). It follows that the diffusion equation is derived to be;

\[
\frac{\partial^2}{\partial x^2} \delta \mu'_v = \frac{1}{(l_v^i)^2} \delta \mu'_v, \quad \text{(S21)}
\]

where the suffix \( i \) corresponds to the region index and \( l_v^i \) is the inter-valley scattering length. The inter-valley scattering should mainly be caused by the edge scattering, so we assume that \( l_v^A = l_v^C = l_v^E = l_v \) and \( l_v^B = l_v^D = \infty \). In analogy with spin Hall effect\(^8\), the relation between the charge current density \( j_x \) in the \( y \) direction, the valley current density \( j_v \) in the \( x \) direction, the electric field \( E \) in the \( y \) direction and the gradient \( \frac{\partial}{\partial x} \delta \mu_v \) of the valley voltage in the \( x \) direction is given by the conductance matrix:

\[
\begin{pmatrix} j_x' \\ j_v' \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & -\sigma_{xy}^{\text{VH}} \\ \sigma_{xy}^{\text{VH}} & \sigma_{xx} \end{pmatrix} \begin{pmatrix} E' \\ \frac{1}{2e} \frac{\partial}{\partial x} \delta \mu'_v \end{pmatrix}, \quad \text{(S22)}
\]

where we assume uniform \( \sigma_{xx} \) and \( \sigma_{xy}^{\text{VH}} \). We neglect the current dispersion effect, so the charge current density is zero except for the region B. In the region B, we assume the uniform charge current density \( j_v^B = j \). At each interface of neighboring regions in Fig. S14, the valley voltage and the valley current density are connected continuously. We assume the valley voltage is zero at the infinitely distant points, namely \( \delta \mu_v^A(x = -\infty) = 0 \)
and $\delta \mu^E(x = \infty) = 0$. By solving equations (S21) and (S22) self-consistently with these boundary conditions, we obtain the following nonlocal resistance formula:

$$R_{NL} = \frac{W}{2l_v} \frac{\left(\sigma_{xy}^{VH}\right)^2}{\sigma_{xx} \left(\sigma_{xx}^2 + \sigma_{y}^{VH}\right)^2} \exp\left(-\frac{L-w}{l_v}\right) \frac{1}{\left(1 + \frac{w}{2l_v}\right)^2 - \left(\frac{w}{2l_v}\right)^2 \exp\left(-2 \frac{L-w}{l_v}\right)}.$$ \hspace{1cm} (S23)

So for $\alpha_{VH} \ll 1$, equation (S23) is approximated by

$$R_{NL} = \frac{W}{2l_v} \left(\sigma_{xy}^{VH}\right)^2 \exp\left(-\frac{L-w}{l_v}\right) \frac{1}{\left(1 + \frac{w}{2l_v}\right)^2 - \left(\frac{w}{2l_v}\right)^2 \exp\left(-2 \frac{L-w}{l_v}\right)}, \hspace{1cm} (S24)$$

and the scaling relation between $R_{NL}$ and $\rho = 1/\sigma_{xx}$ is $R_{NL} \propto \rho^3$.

Further taking the limit of $w \to 0$, equation (S24) becomes

$$R_{NL} = \frac{W}{2l_v} \left(\sigma_{xy}^{VH}\right)^2 \exp\left(-\frac{L}{l_v}\right). \hspace{1cm} (S25)$$

This reproduces the nonlocal resistance formula calculated in the sequential conversion picture in ref. 7.

On the other hand, when $\alpha_{VH} \gg 1$, equation (S23) is approximated by the following formula.

$$R_{NL} = \frac{W}{2l_v} \frac{1}{\sigma_{xx}} \exp\left(-\frac{L-w}{l_v}\right) \frac{1}{\left(1 + \frac{w}{2l_v}\right)^2 - \left(\frac{w}{2l_v}\right)^2 \exp\left(-2 \frac{L-w}{l_v}\right)}. \hspace{1cm} (S26)$$

So the scaling relation between $R_{NL}$ and $\rho$ is $R_{NL} \propto \rho$. But this does not explain the saturation of the nonlocal resistance obtained for a large displacement field (see Fig. 4 in the main text).
Figure S14 | Schematic picture of the nonlocal transport mediated by valley current. Here we assume a Hall bar which is infinitely-long in $x$ direction. Two probes separated by $L$ are attached to both sides of the central channel whose width is $W$. The probe width is $w$. The Hall bar is separated into five regions, A, B, C, D, and E. Charge current is injected into the region B and nonlocal voltage is measured in the region D.
X. Valley Hall conductivity at finite temperature

When elevating the temperature, the valley Hall conductivity decreases. Since carriers are thermally excited from the valence band to the conduction band, the number of valence band electrons decreases while that of the conduction band electrons increases. Because the valence band and the conduction band have opposite sign of the Berry curvature, the valley Hall conductivity, which is calculated from the total Berry curvature decreases. The valley Hall conductivity at finite temperature is calculated from the following equation:

\[
\sigma_{xy}^{\text{VH}}(\mu) = \sum_{\tau_z} \sum_{s_z,\alpha} \frac{e^2}{\hbar} \int \frac{1}{(2\pi)^2} dk_x dk_y \Omega(k, \tau_z, \alpha) f_{\mu}(E_{k,\alpha}), \quad (S27)
\]

where \(\mu\) is the chemical potential, \(\tau_z\) is the valley index (\(\tau_z = -1\) for K and \(\tau_z = +1\) for K'), \(\alpha\) is the band index (\(\alpha = -1\) for the valence band and \(\alpha = +1\) for the conduction band), \(s_z\) is the spin index (\(s_z = -1\) for up spin and \(s_z = +1\) for down spin).

Energy dispersion \(E_{k,\alpha}\), the Berry curvature \(\Omega(k, \tau_z, \alpha)\) and Fermi distribution function \(f_{\mu}(E)\) are given by the following formulas:

\[
E_{k,\alpha} = \alpha \sqrt{\left(\frac{\hbar^2 k^2}{2m}\right)^2 + \Delta^2}, \quad (S28)
\]

\[
\Omega(k, \tau_z, \alpha) = \tau_z \frac{\hbar^2}{m} \Delta \sqrt{E_{k,\alpha}^2 - \Delta^2} \frac{1}{E_{k,\alpha}^3}, \quad (S29)
\]

and \(f_{\mu}(E) = \frac{1}{e^{(\mu-E)/k_B T} + 1}\). \( (S30)\)

At CNP (\(\mu = 0\)), equation (S27) can be simplified as:

\[
\sigma_{xy}^{\text{VH}}(\mu = 0) = \frac{4e^2}{h} F \left( \frac{\Delta}{k_B T} \right), \quad (S31)
\]

where

\[
F(x) = \frac{x}{2} \int \frac{1}{2\xi} \tanh \xi. \quad (S32)
\]

Figure S15 shows the plot of equation (S31). For \(\Delta \gg k_B T\), \(\sigma_{xy}^{\text{VH}}\) saturates at \(4e^2 / h\), and for \(\Delta \ll k_B T\), \(\sigma_{xy}^{\text{VH}}\) decreases to 0. At \(T = 70K\) \((k_B T = 6\text{meV})\), \(\Delta\) dependence of the reduction of \(\sigma_{xy}^{\text{VH}}\) from \(4e^2 / h\) is calculated (see Fig. S15). In Fig. 4 in the main text, the
range of the displacement field $D$ is $D \geq 0.22 \text{V/nm}$, where $\Delta$ estimated from Fig. S7 is $\Delta \geq 10 \text{meV}$. In this range, $\sigma_{xy}^{\text{VH}}$ stays almost constant, and the maximum deviation from $4e^2/h$ is about 10%.

Figure S15 | Calculated valley Hall conductivity at CNP, at finite temperature. The valley Hall conductivity at CNP, at finite temperature is calculated from equation (S31). The valley Hall conductivity depends on the ratio between $\Delta$ (half of the band gap) and $k_B T$. Inset: Plot of the reduction of $\sigma_{xy}^{\text{VH}}$ from $4e^2/h$ ($\delta \sigma_{xy}^{\text{VH}} = 4e^2/h - \sigma_{xy}^{\text{VH}}$) as a function of $\Delta$ at $T = 70 \text{K}$. 
References


