Enhanced optical trapping via structured scattering: Supplementary information

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S1: Physical limit to trap stiffness

It is well known that the magnitude of optical forces is limited by the momentum flux of the light. This momentum flux is equal to the radiation pressure force on full absorption, and is given by $n_m P/c$, where $P$ is the incident power, $n_m$ the medium refractive index, and $c$ the speed of light\(^1\). Optical forces are commonly described with the normalized force $Q$, which is the fraction of the radiation pressure imparted on the particle. Provided the field only interacts once with the particle, this parameter has an upper limit of 2 which is reached for complete reflection of a collimated field. However, this is a propulsion force which always points in the direction of optical propagation. When considering a trapping application, it is most useful to consider the maximum restoring force. The optical force is determined from the difference in the optical momentum before and after the interaction; and since the momentum of the incident light is independent of the particle position, this force can only change with particle position over a maximum range of $\Delta Q = 2$.

The most stable optical trap is reached when this range of applicable force is centred about zero, such that the maximum useful trapping force is limited to $Q = 1$.

In addition to this well known limit, there is also a physical limit on the trap stiffness which has not, to our knowledge, previously been discussed. The trap stiffness is defined by the change of the trapping force with the position of the trapped particle, $k = -\frac{dF}{dx}$. This is constrained because radiation pressure constrains the maximum force $F$, while the optical wavelength sets the minimum length scale over which a propagating optical trapping fields can significantly change. The main text presents a simple scenario where a beamsplitter is trapped by counter-propagating fields. Displacement of the beamsplitter shifts the relative phases of these fields such that the output intensities are modulated by interference. This can be stably trapped since beamsplitter displacements lead to a relative phase shift on the incident fields. In the limit of normally incident light on a 50/50 beamsplitter, this relative phase shift is given by $4\pi n_m x/\lambda$, with $x$ the displacement around the stable trapping point and $\lambda$ the vacuum wavelength. The resulting mean power imbalance of the two output fields is then given by

$$\langle \Delta P \rangle = P \sin(4\pi n_m x/\lambda),$$

with a resulting optical force of

$$F = -\frac{n_m P}{c} \sin(4\pi n_m x/\lambda),$$

and trap stiffness of

$$k = \frac{4\pi n_m^2 P}{c\lambda}.$$  \(3\)

When compared to optical trapping experiments, this is an extremely high trap stiffness. When using 1064 nm light in water, for instance, this corresponds to a stiffness of 70 mN m\(^{-1}\) W\(^{-1}\), which is 18 times higher than ever demonstrated in an experiment\[^2\]. Here we argue that this stiffness represents the upper limit on achievable optical trap stiffness.

We first note that Eq. 3 corresponds to a complete reversal of optical momentum ($\Delta F = 2n_m P/c$) over a distance scale of ($\Delta x = \lambda/2\pi n_m$). This is the greatest possible change in momentum over a the shortest length scale relevant to the light, distance over which 1 radian of phase accumulates. As such, it is somewhat intuitive that this should represent

\[^1\]The momentum flux of light in a medium has been highly controversial, with Abraham and Minkowski separately deriving it to be $P n_m$ and $n_m P/c$ respectively, and with both forms confirmed by experiments\[^1\]. The different forms have been shown to describe different physical quantities, as discussed in detail in Ref. \[^1\]. It is the Minkowski form $n_m P/c$ which is appropriate to this problem, as this conserves kinetic momentum in light-matter interactions.

\[^2\]
an upper limit on optical trap stiffness. Below, we further show that a hypothetical experiment which could surpass this limit must also violate the lower bound on displacement precision.

In optical trapping applications, position measurements are performed by measuring the distribution of output power. This distribution depends on the outgoing momentum flux which determines the applied optical force $F$. In general, one can interpret position measurements as force sensing experiments, with the position estimated as

$$ x = \frac{F}{\kappa}. \quad (4) $$

The associated measurement uncertainty is then given by

$$ \delta x = \frac{\delta F}{\kappa}. \quad (5) $$

Consequently, the largest achievable trap stiffness is constrained by the quantum limits to both position sensitivity and optical force noise, with

$$ \kappa_{\text{max}} = \frac{\delta F}{\delta x_{\text{min}}}. \quad (6) $$

Here we consider trapping with coherent light, which corresponds to the output of a noise-free laser. Although termed noise-free, the momentum of this field fluctuates with field radiation pressure shot noise of $\delta F = (\nu_m h / \lambda) N^{1/2}$, where $h$ is Planck’s constant and $N$ the mean number of photons that are incident within the time window considered. In general, not all of this noise need be transferred to the particle. However, the trap stiffness is maximized when this radiation pressure shot noise is mapped fully onto the particle (see Eq. 6). Additionally, the lower limit on measurement precision (often called the standard quantum limit) is given by

$$ \delta x_{\text{min}} = \frac{\lambda}{4\pi n m N^{1/2}}, \quad (7) $$

as derived separately in Refs. [3] and [4]. Substituting these into Eq. 6, we find the upper limit to trap stiffness to be given by

$$ \kappa_{\text{max}} = \frac{4\pi n^2 m h N}{\lambda^2} = \frac{4\pi n^2 P}{c\lambda}, \quad (8) $$

where we have used $P = Nh c / \lambda$. This upper limit corresponds exactly to the trap stiffness achieved with the beamsplitter, as described in Eq. 3. This is unsurprising as the scheme allows position sensing at the limit of Eq. 7 via a simple measurement of the output power imbalance, while also being subject to the full radiation pressure shot noise. Consequently, the trap stiffness $\kappa$ derived for this system corresponds to the highest stiffness which is achievable for propagating coherent light.

One may consider that this limit could be surpassed by using a noisy laser, as $\kappa_{\text{max}}$ appears to increase with the noise in the applied force (see Eq. 6). This is not the case, as a noisy light source also increases the minimum resolvable displacement, with Eq. 7 only achievable in the limit of ideal detection with perfectly coherent light. However, it may be possible to surpass this limit on stiffness using near-field optics such as plasmonic trapping, using multiple light-particle interactions such as a trap within an optical cavity, or by using non-classical states of light for which the position sensitivity can in principle surpass that of Eq. 7.

**S2: Optical trapping forces**

The trapping forces achieved with ENTRAPS are modelled using an electromagnetic treatment based on extended Mie scattering. Here we provide a brief introduction to optical forces in the Mie scattering regime; for a more detailed discussion we refer readers to Ref. [5].

If the trapped particle is much smaller than the optical wavelength, the scattering can be approximated as dipole scattering. Dipole scatter is a special case of the more general Mie scattering, and consequently can be analyzed within the more general framework. However, the dipole scattering force is usually expressed in terms of the gradient and radiation pressure force contributions, which is an approach that cannot be extended to the more general case of Mie scattering.

In the usual treatment of a dipole scattering particle centred at the origin, the optical field $E$ at the particle induces an oscillating dipole with polarization $p = \alpha E$, where $\alpha$ = is the polarizability of the particle[6]. This can then be used
to derive two separate contributions to the optical force. The optical gradient force is proportional to the polarization $\alpha$ and the local gradient in intensity, and pulls particles toward the maxima in intensity; while radiation pressure exerts a non-conservative pushing force on the particle which is proportional to the intensity and $\alpha^2$. As such, the optical force is fully determined by the local amplitude and gradient of the field at the particle[6].

To calculate the optical force with a Mie scattering particle, we do not consider the field locally around the particle, but instead consider the far-field angular profile of the field. This is not truly distinct from an analysis of local fields, as the far-field angular profile of the optical field is sufficient to completely determine the field at every point in space, provided we consider only propagating fields and not localized near-field effects. Within this framework, the incident field is decomposed into vector spherical harmonic (VSH) functions,

$$E(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_{n,m} \Psi_{n,m}(\theta, \phi) + \beta_{n,m} \Phi_{n,m}(\theta, \phi)$$  \hspace{1cm} (9)

Here $\Psi$ and $\Phi$ are vector functions with orthogonal polarizations, and $n$ and $m$ are the principal and azimuthal indices. Representation of a three-dimensional vector field strictly requires three separate vector functions; however, these functions are defined in the far-field where we assume the radial polarization component to be zero. Each of the VSH functions is an eigenmode of the scattering interaction when the light is incident on a homogeneous spherical particle which is centred at the origin. Consequently, Mie scattering can be represented as a matrix multiplication between the scattering T-matrix and the VSH coefficients $\alpha$ and $\beta$,

$$\left( \begin{array}{c} p \\ q \end{array} \right) = T \times \left( \begin{array}{c} a \\ b \end{array} \right) ,$$  \hspace{1cm} (10)

where $p$ and $q$ are the VSH coefficients of the scattered field. Since the VSH functions are eigenmodes of the scattering process, the T-matrix is diagonal.

Dipole scattering is achieved in the $n = 1$ modes of $\Phi$, with $\Phi_{1,-1}$, $\Phi_{1,0}$ and $\Phi_{1,1}$ representing the dipole scatter from left circular, axial, and right circular polarized light respectively. These are also the only modes with a non-zero contribution to the field at the origin, so the coefficients $b_{1,m}$ can be evaluated directly from the field amplitude at the origin (similar to the usual dipole scattering treatment). Consequently, the dipole approximation is expressed in the Mie scattering treatment as $T = 0$ for $n > 1$, with the dipole scattering modes alone providing the radiation pressure force, and interference between these modes and all other modes providing the gradient force.

More generally, the scattered field includes contributions in all of the VSH modes. For any given scattering system the applied force is given by the change in optical momentum of the field. The net momentum flux of the optical field, and hence its radiation pressure force, is given by

$$F_{RP} = \int \epsilon / 2 |E(\theta, \phi)|^2 \hat{u} \ dA,$$  \hspace{1cm} (11)

with $\hat{u}$ the normal unit vector. This calculation forms the basis of all of the force calculations presented in this paper. Such calculations are carried out here using functions from the Optical Tweezers Toolbox[7].

For large particles, the dipole approximation fails for two reasons. Firstly, the scattering from the $\Phi_{1,1}$ diverges from the predictions of dipole scattering, and the scattered field ceases to scale with the particle volume. This leads to a decline in trapping force relative to the predictions of dipole scatter. The onset of this decline is generally viewed as the point where dipole scatter begins to break down. However, the physics of the trapping force remains essentially unchanged from the simpler dipole scattering case, which is why the qualitative characteristics of the trapping force persist when trapping particles of similar size to the wavelength. Secondly, and more importantly for this work, the scattered field for larger particles includes contributions in the higher order VSH modes. These modes have qualitatively different spatial structure, with both positive and negative amplitudes at different angular positions. This allows the interference to have non-trivial spatial structure that is not possible for $n = 1$ modes, and consequently allows new characteristics in the applied optical force.

Outside of the dipole scattering regime the higher order VSH modes dominate the scattering interaction. Even for relatively small particles (>650nm), dipole scattering modes ceases to be the dominant form of scatter, with the orthogonally polarized $\Psi_{1,m}$ modes coupling more strongly than the $\Phi_{1,m}$ modes. However, these modes are still first order ($n = 1$), and therefore show considerable similarities to the dipole scattering modes. As the size increases, the higher order modes become more important. For particles larger than 1800 nm one finds that the first order modes cease to dominate the scattering terms in the T-matrix, and the particle couples most strongly to higher order modes. As the particle size increases, the most strongly scattering VSH modes become higher and higher order. For instance, a 10 μm diameter silica sphere interacts most strongly with the $n = 38$ modes.
In this regime, a focused Gaussian achieves a trap stiffness that approximately scales inversely with the particle size. The Gaussian is homogeneous over the largest possible range of incident angles, and consequently places most of its power into the lowest order $n = 1$ modes. This is an efficient strategy for dipole scattering particles (which couple most strongly to these modes) but becomes increasingly inefficient as the particle size is increased. By structuring the incident field, one can controllably populate light into the strongly interacting higher order modes. However, simply populating light into these modes is not adequate to achieve strong trapping. Strong trapping requires not only strong scattering, but also efficient interference between the different modes to maximize the optical force (see Eq. 11). Because the high-order modes exhibit considerable spatial structure, the interference can vary strongly with angular position. With appropriate structuring, this can be used to achieve well controlled interference in the output field. This spatial control over the interference pattern is not possible with $n = 1$ modes, and therefore the resulting force can be engineered with quantitatively different characteristics.

The experiments presented here make use of the control offered by these higher order modes. By carefully tuning the incident phase, we both populate the strongly interacting modes and ensure that the resulting interference effectively converts the particle into a microscale beamsplitter, which allows order-of-magnitude enhancements in trap stiffness.

**S3: Algorithm to locally optimize trapping profiles**

To calculate the trapping profile for ENTRAPS, we performed an iterative optimization routine to maximize the trap stiffness. This algorithm naturally tends to locate profiles which exhibit ENTRAPS for large particles, as these can achieve order-of-magnitude enhancements in trap stiffness. ENTRAPS characteristics can be observed over the full range considered here (see Fig. 1). However, the algorithm does not specifically select profiles that achieve ENTRAPS, so it can in principle converge on a profile that does not follow beamsplitter-like characteristics.

The algorithm utilizes code from the Optical Tweezers Toolbox (OTT) to calculate the scattered field and the resulting optical forces[7] (see Section S2). This framework requires the incident optical field to be decomposed into an expansion of vector spherical harmonics, given by the coefficients $a_i$ and $b_i$. Each of these spherical harmonics are described by integer orbital and angular momentum parameters $n$ and $m$ respectively, with the constraint that $|m| \leq n$. It is necessary to truncate this expansion at a maximum value for $n$, referred to as $N_{max}$. Accurate calculations require $N_{max}$ to be sufficiently large to capture all details of the field, though the algorithm can become computationally very inefficient for a large total number of modes. For a given value of $N_{max}$, the total number of modes included in the calculation is $2N_{max}^2 + 4N_{max}$. We have used this algorithm with $N_{max} = 65$, which corresponds to inclusion of 8710 modes. This expansion is sufficiently large to avoid numerical errors in the calculate forces. To ensure that the calculations are accurate, the force exerted with the calculated phase-profiles were also calculated up to $N_{max} = 100$, which includes of 204000 modes. This larger expansion is not used in the algorithm as it requires excessive computer memory and finds similar results.

We find it convenient to change between the spherical harmonic basis, defined by $a$ and $b$, and the angular field distribution $E(\theta, \phi)$ at angular coordinates $\theta$ and $\phi$. We convert basis using matrices $A_p$ and $B_q$, which were introduced in the quadrant detection package[33]. These matrices include implicit angular dependence, such that they can be used to convert between bases using the matrix multiplications

$$E(\theta, \phi) = A_p \times a + B_q \times b$$

$$a = A_p^T \times (E(\theta, \phi) \cdot dA)$$

$$b = B_q^T \times (E(\theta, \phi) \cdot dA)$$

where the superscript $T$ denotes a complex transpose and $dA$ is the area element calculated in the direction $(\theta, \phi)$. To control the trapping light, we define a phase plate with 8000 modes $\psi_i$, with the mode decomposition defined by the spatial frequencies of the modes. In this case $i$ denotes the mode number, and the phase plate can be fully determined by the coefficients $c_i$ of these modes. The phase plate is applied to a Gaussian trapping field in the angular field basis, and the field converted back to the spherical harmonic basis. The trap stiffness is then evaluated in the manner presented in the OTT, though this calculation was modified to avoid repeated recalculations of the same variables. The trap stiffness defines the fitness $F$ of the phase plate, which is the variable to be maximized in this algorithm.

To maximize the trap stiffness, we first calculate the change in trap stiffness for small changes in each of the coefficients $c_i$ of the phase plate. Once this is known, we define a collective mode $\Psi$ of the phase plate which can most strongly improve fitness, defined as

$$\Psi = \sum_i \delta F \cdot c_i \psi_i.$$  

(15)
Figure 1: Polar plots of the transmitted intensity when using the calculated holograms for each of the particle sizes considered in Fig. 3a of the main text. Blue curves and red curves respectively show the intensity when the particle is centred, and displaced 150 nm right. The characteristic beamsplitter-like trapping of ENTRAPS is observed over the entire size range. For 1 μm particles this effect is somewhat weak, and provides minimal enhancement. As the size increases, the interference fringes become more well defined and the ENTRAPS effect more pronounced. Further increasing the particle size results in many more fringes, and an increasingly complex transmission pattern.
This collective mode is added to the phase plate with varying amplitudes until a local optima in fitness is found. The phase plate is then updated, and the algorithm then repeats as many times as necessary. Each iteration of the algorithm deterministically deforms the phase plate toward a local optima. Our investigations have confirmed that when starting with different initial values, different locally optimal solutions are found. Consequently, the algorithm is not guaranteed to locate the global optimum which provides the best possible trap stiffness, though it does provide substantial improvements in trap performance. To the best of our knowledge, there is no existing method which can find the global optima for phase-only control of the light.

This algorithm has been used to calculate enhanced trapping profiles, both for phase-only control of the incident light and with phase and amplitude control. Amplitude control was simply included in the above algorithm in the same way as phase control, with an amplitude plate perturbing the default Gaussian intensity profile. Using this, we confirmed that full wavefront control can allow stronger trap stiffness than phase-only control. We note, however, that the Optical Eigenmode method can find the global optimum for the case of combined amplitude and phase control[9, 10], so our algorithm is of limited relevance to experiments in which full wavefront control is achievable. Our algorithm is instead relevant to holographic optical tweezers, in which phase-only control can be readily achieved.

The algorithm, as it is currently written, is computationally very intensive. To calculate a single trapping phase profile, over 10 iterations and with $N_{\text{max}} = 65$ currently takes 120 hours on a 4-core PC with 16GB of RAM and 3.4 GHz processors. An improved algorithm would be highly valuable if it could locate global optima and/or converge more efficiently, as this would allow a more comprehensive study of the capability of such profiles and also make the strategy more accessible to the optical trapping community.

Mapping the phase profile into the optical trap

To generate the phase profiles calculated with the algorithm above, we used an SLM which was mapped to the back-focal plane of the trapping objective. We used Abbe’s sine condition to map the phase profile from the SLM to the angular coordinates used in the calculations (see Fig. S2). When the objective focuses the incoming field, the phase shift on the SLM is mapped to the far-field profile from the objective focus. This maps the SLM coordinates to far-field angular coordinates $(\theta, \phi)$ around the focus. This can be described simply if the SLM coordinates are represented in polar form $(\rho, \phi)$. In this case, the mapping preserves $\phi$ unchanged, while the radial position $\rho$ maps to the angular position $\theta$ according to Abbe’s sine condition: $n m \sin(\theta) = \rho/C$, with $C$ the radius of the objective back aperture.

Effect of numerical aperture

As is the case for most approaches optical trapping, the numerical aperture (NA) of the objective can be expected to be an important parameter in determining the effectiveness of ENTRAPS. While we do not present explicit calculations here,
we expect that the achievable trap stiffness would increase with increasing NA. As discussed in Section S2, large particles couple strongly to higher order modes, and the structured trapping solutions make use of this. The number of accessible modes is limited by the objective NA. An increase in NA should increase the utility of such modes and therefore the possible stiffness.

S4: Experimental data

To verify the predictions of our model, we characterized ENTRAPS trapping with silica particles of four different diameters: 3.48 µm (Bangs Labs), 5.09 µm (Bangs Labs), 7.75 µm (Cospheric), and 10.0 µm (Microspheres-Nanospheres). In all cases we assumed the refractive indices of water and silica to be 1.33 and 1.46 respectively. For each of these four particle sizes, the particles were trapped with both Gaussian and structured traps, and the resulting thermal vibrations measured on a position sensitive detector (PSD) at the back-focal plane of a high NA condenser.

Unlike a quadrant detector, the PSD provides a signal which is proportional to the position of the laser intensity centroid on the detector element, and this is directly proportional to the applied optical force [12]. Provided the particle stays within the linear region where $F = -\kappa x$ (as was the case here), this also provides an accurate estimate of particle displacement.

For each particle size, the thermal motion was measured and transformed into spectra. The thermal spectra was determined by averaging over 4 or more individual sets of data to reduce the noise. The enhancement in trap stiffness was then determined by fitting the corner frequency to the measured spectra, as described for Fig. 3b of main text. In each of these cases, the corner frequency was fitted using the “nlinfit” function in Matlab, while the uncertainty was estimated using the covariance matrix with the “nlparci” function. Likewise, the enhancement in SNR was determined from these spectra from the increased signal amplitude. The main text shows only the spectra from 7.75 µm particles. Here in Fig. S3 we present the spectra measured for all four sizes, which clearly shows a clear enhancement via ENTRAPS in all cases. These enhancements are specified in Table 1.

In addition to these spectra, force-displacement curves were also measured for 10 µm and 7.75 µm particles to allow quantitative comparison of the experiments could with the predictions of theory (Fig. S4). The input trapping power was reduced to approximately 1 mW such that the particles were immobilized on the glass coverslip by gravity, and the detector response was measured as the particle was scanned over the laser focus. As noted above, the PSD estimates the applied optical force, so this measurement maps the force at different displacements. The voltage output from the PSD was calibrated into normalized force units by fitting one full force-displacement curve for the Gaussian trap to the curve predicted with the Optical Tweezers Toolbox (Fig S4a, 10 µm particle), which shows excellent agreement. The force could then be quantified without any fitting parameters for the ENTRAPS profile. Good agreement with the theoretical prediction is obtained for both 10 µm and 7.75 µm particles (Figs. 4b and 4c), though the 10 µm curve shows some asymmetry which may result from movement of the particle on the coverslip.

The movement of the particle could in principle be removed by firmly fixing the particle on the coverslip. This could be simply achieved by leaving the particles to settle on the coverslip for a long time. However, we found that the force-displacement curves measured with firmly fixed particles did not follow the trends predicted by theory and measured with gravitationally bound particles. We postulate that the scattering properties of the particles were influenced slightly by the fixing, possibly due to deformation of the surface.

### Table 1: The characteristics of the spectra shown in Fig. S3. The corner frequencies and enhancements are shown here, together with the predicted enhancement (as shown in Fig. 3 of the main text).

<table>
<thead>
<tr>
<th>Diameter</th>
<th>ENTRAPS corner</th>
<th>Gaussian corner</th>
<th>Enhancement</th>
<th>Predicted enhancement</th>
<th>SNR improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.48 µm</td>
<td>132±7.5 Hz</td>
<td>24.7±2.2 Hz</td>
<td>5.33±0.57</td>
<td>7.5</td>
<td>22.8</td>
</tr>
<tr>
<td>5.09 µm</td>
<td>132.3±5.6 Hz</td>
<td>10.0±1.5 Hz</td>
<td>13.3±2.1</td>
<td>14.8</td>
<td>111</td>
</tr>
<tr>
<td>7.75 µm</td>
<td>73.2±2.8 Hz</td>
<td>2.66±0.38 Hz</td>
<td>27.5±4.1</td>
<td>26.1</td>
<td>249</td>
</tr>
<tr>
<td>10.0 µm</td>
<td>68.4±4.9 Hz</td>
<td>3.4 ± 0.7 Hz</td>
<td>20.1±4.4</td>
<td>24.1</td>
<td>107</td>
</tr>
</tbody>
</table>

2Additionally, the thermal spectra extracted from a single set of data follows non-Gaussian probability distributions around the mean value at each frequency. This can lead to systematic errors when fitting the data to theory [11]. Averaging the spectra reduces this to a Gaussian distribution with the same mean via the central limit theorem.
Figure 3: Experimentally measured spectra of motion for all four particle diameters: 3.48, 5.09, 7.75, and 10.0 µm, as indicated for each column. Top row: spectra measured with Gaussian traps, bottom row: spectra measured with ENTRAPS. The corner frequency is indicated within each plot. Insets: the phase hologram which was used to achieve ENTRAPS. In every case, the use of ENTRAPS provided a clear increase in trap stiffness. Additionally, it improved the SNR, as can be seen from the increased amplitude in the characteristic 1/f² roll-off region. For clarity, all parameters associated with these plots are tabulated in Table 1.

Figure 4: Force-displacement curves were measured and compared to theory predictions. a The Gaussian trap shows excellent agreement between theoretical predictions (thick line) and the measurement from the PSD (dark curve); data taken with 10 µm particles. This profile was used to normalize the signal from the PSD into the unit $Q$, which is defined as the proportion of the radiation pressure that is applied as force. The shaded regions indicates the axes of b, which compares the force-displacement curve for a Gaussian trap to ENTRAPS. The increase in trap stiffness is evident here, with ENTRAPS drastically increasing the trap gradient over a small range. c Similar results are also found for 7.75 µm particles, with the increase in stiffness over use of the Gaussian trap clearly visible. The theory curves are calculated with no fitting parameters.
Figure 5: Dependence of the trap stiffness on (a) particle diameter and (b) refractive index, for the 10 µm ENTRAPS hologram. The particle is centered at the focus, and the trapping field is the same as that used in the main text. The curve is dashed where the trap stiffness falls below zero, as this denotes an anti-trap feature with the particle repelled from the beam center. The trap stiffness can be seen to vary strongly with both the particle diameter and refractive index.

S5: Unique characteristics of ENTRAPS

Particle tolerances

The optical forces which arise from ENTRAPS rely on precise knowledge of the scattering matrix of the particle, and can perform poorly if the particle size or refractive index differs considerably from the assumed parameters. This leads to resonance-like features, though we note that ENTRAPS need not feature any storage of optical energy as required for an optical resonator. For instance, Fig. S5 shows the trap stiffness predicted for the ENTRAPS trap of 10 µm spheres as the particle radius or refractive index is varied. A change in particle diameter of 0.25 µm halves the trap stiffness, and a change of 0.4 µm makes the trap unstable (Fig. S5a). Similarly, changing the particle refractive index by 0.04 halves the trap stiffness, and changing it by 0.066 makes the trap unstable (Fig. S5b). It is perhaps no coincidence that a change in diameter of 0.4 µm and a change in refractive index of 0.066 both alter the optical path length through the microsphere by approximately half an optical wavelength, and consequently this can reverse the sign of the scattering fringes. These tight tolerances mean that the particle diameter and refractive index must be accurate to within 1% for the calculations to accurately predict the experimental trapping conditions. Our experimental observations verify that ENTRAPS introduces tight tolerance on particle size, as we observed that particles with slightly different sizes did not provide a strongly enhanced trap stiffness.

However, a simple rescaling of the ENTRAPS phase profile can partially compensate a change in particle diameter (see Fig. S6). For a small change in particle diameter, the high trap stiffness can be approximately recovered by simply rescaling the phase profile along the x axis while leaving the y dimension unchanged. This broadens the tolerance in diameter by an order of magnitude, and therefore makes experimental demonstration far easier. It is important to note that this rescaling should be applied along the angular profile of the phase hologram in the far-field, rather than in the plane of the SLM; with Section S3 describing the conversion between SLM coordinates and far-field angular coordinates.

Trapping features

In addition to this, we also found that a phase shift on the incident light could transform the system between stable trapping with the particle confined at the focus, and anti-trapping with the particle repelled from the focus. The force-displacement curves shown in Fig. S7 were recorded for the same 10 µm particle and the same trapping profile, but at different axial displacements. When the particle was located at the focus, strong ENTRAPS trapping was achieved with a trap stiffness enhancement of approximately 20. However, raising the particle by 2.54 µm transformed the force into a strong anti-trap that would repel the particle from the optical axis.

Another unusual feature is the existence of multiple stable trapping sites, which can also be seen in Fig. S7a. Multiple trapping sites result when the trapping force oscillates with position. This was predicted theoretically, and also follows
Figure 6: In this work, we have relied on accurate precalculation of the scattering profile to achieve ENTRAPS. This introduces tight tolerances in the particle diameter, as shown in Fig. S5. However, small changes in the particle diameter can be compensated by rescaling the $x$ axis of the phase hologram (plot a), as shown for the 10 $\mu$m particle hologram. This allows the size tolerance to be broadened by over an order of magnitude, which makes experimental demonstration far easier. Large regions of this plot show no stable trap (dark blue, $\kappa = 0$); in these regions the trapping phase is mismatched to the particle size such that it is repelled from the optical focus ($\kappa < 0$). Higher order trapping sites are evident where the change in the transmitted phase approaches $2\pi$ such that stable trapping can once again become possible. b We tested the dependence of the stiffness enhancement on the rescaling parameter for a 9.8 $\mu$m diameter particle, and compared the data to theory. This shows good agreement around the over the peak trapping range, though at very small rescaling parameter we observed stable trapping even when this was not predicted. This is because the trapping field features multiple trapping sites, and when the central trapping site becomes unstable the particle can remain in secondary trap sites (see Fig. S7).
Figure 7: Force-displacement curves measured with the PSD, with a 10 µm particle (a) near the focal plane, and (b) 2.54 µm above the focus. The trapping field is the same as used for the results in section S4. Near the focus the calibration curve shows a strong trapping feature, while above the focus this changes into an anti-trap feature. This data also shows some asymmetry between positive and negative displacements in x, which suggests that optical aberrations were not fully compensated for this set of data.

from the simplified beamsplitter analogy discussed in Section S1. Multiple trapping sites were observed commonly when using the ENTRAPS profile, both for strongly and weakly trapped particles. For instance, the anti-trapping profile shown in Fig. S7b features five stable trapping sites, though none show the strong trapping which is characteristic of the main trapping site in ENTRAPS.

These features are completely foreign to typical optical trapping experiments with Gaussian laser profiles. However, we note that the strict tolerances in this method are also found in experiments using anti-reflection coated particles[2]. That approach can be considered as the conjugate of ours; here, we engineer the trapping light to suit the scattering properties of a specific particle, while there the particle scattering properties are engineered to better suit the trapping light. Although both methods introduce strict tolerances, the underlying physics of ENTRAPS differs significantly from use of anti-reflection coated particles. It would be interesting to explore whether these two methods could be combined.

References

References


