Supplementary Information


We provide here supplementary materials for our Article, including further analysis of the CNOT gate by quantum process tomography, and additional information regarding the calibration of thermal phase shifters.

Quantum Process Tomography As a demanding test of the reconfigurability of our circuit we used the eight phase shifters to perform all of the measurements required for quantum process tomography (QPT)\textsuperscript{1–3} of the central entangling CNOT gate. QPT is a well-known, complex and resource-intensive procedure and as such provides a strong test of the reconfigurability of this circuit, as well as providing a complete characterization of the CNOT gate. By setting appropriate voltages to phase shifters \( \phi_{1-4} \) we prepared 16 separable, linearly independent input states \( \{ \rho_i^{(\text{in})} \} = |\psi_i\rangle \langle \psi_i | \) at the input of the CNOT gate, where \( |\psi_i\rangle = |\nu_A\rangle \otimes |\nu_B\rangle \) and \( |\nu\rangle = (|0\rangle, |1\rangle, |+\rangle, |+\rangle) \). For each \( \rho_i^{(\text{in})} \), the density operator of the state generated by the CNOT gate \( \rho_i^{(\text{out})} = \sum_{m,n=0}^{d^2-1} \chi_{mn} A_m \rho_i^{(\text{in})} A_n^\dagger \) (where \( A_m \) are the Kraus operators and \( \chi \) is the process matrix) was reconstructed by QST as before, using phase shifters \( \phi_{5-8} \) to perform each of the 16 measurements.

The process matrix \( \chi \), which completely and uniquely describes the process in question, was then reconstructed according to the maximum likelihood technique comprehensively described in\textsuperscript{4} (Fig. 1). The process fidelity \( F_P = \text{Tr}(\chi_{\text{ideal}} \chi_{\text{exp}}) \) was measured to be 0.841±0.002. This is comparable with the process fidelity of 0.87 previously measured using an equivalent bulk-optical circuit\textsuperscript{4}. The average fidelity\textsuperscript{5}, defined as the state fidelity between actual and ideal output states, averaged over all possible inputs, is 0.873±0.001. Here error was determined by a Monte-Carlo approach, assuming Poissonian photon statistics. Due to the number of measurements required we operate with a shorter integration time than that used for state tomography. As well as imperfections in the fabrication of the CNOT gate itself, finite accuracy in the calibration of the phase shifters also contributes to the subunit fidelity observed.

Details of Heater Calibration Each phase shifter can be seen to occupy a particular MZ interferometer. Referring to Fig. 2, phase shifter S1 occupies an MZ formed by directional couplers C2 and C3, S6 occupies an MZ formed by C4 and C5. S2 and S5 can be seen to occupy a single lossy MZ formed by C1, C3, C6 and C4, where C1 and C6 cause loss. S3 occupies an MZ formed by C7 and C8, and S4 occupies an MZ formed by C8 and C9. Similarly for S7 and S8.

We begin by injecting light into waveguide W2, placing detectors at outputs W7 and W10, in order to measure an interference fringe from S1 - sweeping the voltage between 0V and 7V. There is some loss prior to detection due to C1 and C10-C12, however this only influences the amplitude of the measured fringe - not its phase etc. We take a similar approach for S6, injecting light into W1 and placing detectors at W8 and W9. Here we effectively have lossy input to the MZ due to C1.

When calibrating S2 and S5, there is some ambiguity in the choice of \( \phi(V = 0) \) as both shifters occupy the same MZ. We therefore choose (arbitrarily) that \( \phi_{S2}(0) = 0 \) and then calibrate S5 in the normal way.

C7, C8 and C9 form an interferometer containing S3 and S4. If bright laser light is injected at waveguide W4, the

![Fig. 1: Quantum process tomography of a maximally entangling gate. a Ideal process matrix of the CNOT gate in the centre of the circuit shown in Fig. 1. Imaginary part is zero. b Real and c imaginary parts of the measured process matrix.](image-url)
intensity measured at a detector placed at the upper output port of C9 will vary as

\[ I \propto \frac{1}{2} \left[ 1 + \sin(\phi_{S3}(V_{S3})) \cdot \sin(\phi_{S4}(V_{S4})) \right], \]  

(1)

where \( \phi_{SN} \) and \( V_{SN} \) are the phase shift and voltage across shifter \( SN \), respectively.

\( \phi_{S4} \) dictates the amplitude of the interference fringe measured when \( V_{S3} \) is varied. If \( \phi_{S4} < \pi \), the phase at any point on the measured fringe will correspond exactly to \( \phi_{S3} \). However if \( \phi_{S4} > \pi \), the reconstructed fringe will be flipped and its phase will be shifted by \( \pi \). As we do not initially have calibration data for either S3 or S4, we can only calibrate the phase-voltage relationship of S3 and S4 together modulo \( \pi \). In practice the different self-consistent combinations of phase offset are indistinguishable, and do not affect the experimental outcome. S7 and S8 are calibrated using the same approach.

We used one-photon interference fringes to estimate the overall accuracy with which phase can be controlled. Having completely calibrated the system, we measured interference fringes with 11 points uniformly separated in phase (rather than voltage) for each heater, with \( \phi \in [0, 2\pi] \). We then fit sinusoidal curves of the form

\[ I' + I \sin(\omega \phi + \varphi) \]

(2)

to these datapoints, where \( \omega \), \( I \) and \( I' \) are parameters of the fit. Ideally, \( \varphi = N \frac{\pi}{2}, N \in \mathbb{Z} \), where \( N \) depends on the configuration of the interferometer in question and which output port is used. The mean value of \( |\varphi \mod \frac{\pi}{2}| \) across all eight heaters was found to be 0.05 radians.

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