Onsager's Wien effect on a lattice

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1. Supplementary Discussion: Characteristic length scales

There are multiple length scales relevant to the physics of an electrolyte in addition to the Bjerrum length

$$\ell_T = \frac{q^2}{8\pi \epsilon k_B T}$$

and the field length

$$\ell_E = \frac{k_B T}{qE}$$

the Debye length

$$\ell_D = \sqrt{\epsilon k_B T V_0 / q^2 n},$$

above which the Coulomb interaction is screened through the formation of a correlated charge cloud ($V_0$ is the volume per lattice site); the mean charge separation,

$$\ell_n = (V_0/n)^{1/3};$$

a minimum length $a$ for the separation of two charges, as well as the macroscopic scale $L$ of our numerical simulation box. The main result of Onsager’s theory, equation (1), is valid in the limit that $\ell_D$ and $L$ exceed $\ell_T$ and $\ell_E$, which in turn must exceed the microscopic length $a$. The small field limit exposing the linear field dependence requires the rigorous separation of scales: $L \gg \ell_D \gg \ell_n \gg \ell_E \gg \ell_T \gg a$, which can only be approached for temperatures well below the temperature scale set by the chemical potential, $\mu$. In Fig. 1b we show the evolution of these lengths with temperature for our lattice electrolyte. The window of application available to our simulations, in which the Wien effect can be measured, is bounded from above by the crossover, $\ell_D < \ell_T$ and from below by the finite size limit, $L < \ell_n$, below which we have, on average fewer than one pair of particles in our simulation box.

2. Supplementary Discussion: Reduced units

In the main text, reduced units for corresponding states of electrolytes were introduced:

$$\mu^* = \frac{\mu}{U_0}, \quad T^* = \frac{k_B T}{U_0}, \quad E^* = \frac{U_E}{U_0}.\quad (1)$$

In Supplementary Table 1 we list appropriate parameters for Dy$_2$Ti$_2$O$_7$ spin ice at 0.5 K, for water ice at 253 K, for pure liquid water at 298 K and several other materials. We added phosphoric acid, since it is the strongest known protonic conductor and should exhibit a screened version of Wien effect (with defect concentrations $\sim 0.07$). Note that the relative permittivity for water ice is not the bulk value but instead is the high frequency value, as advocated by Onsager and Dupois$^2$, from which other parameters are taken. The low frequency value was assumed for liquid water and phosphoric acid. For the spin ice/lattice electrolyte parameters, the corresponding charge concentration of order $n \sim 10^{-5}$ is possible to handle numerically, while being sufficiently small to represent the low density limit in general. Likewise, Debye–Hückel and Bjerrum corrections for this parameter set are small but non-negligible. Hence it is a suitable system on which to test the expectations outlined above.
### Supplementary Table 1: Parameters for the double equilibria

<table>
<thead>
<tr>
<th>System</th>
<th>a [Å]</th>
<th>q/e</th>
<th>εr</th>
<th>T [K]</th>
<th>µ∗ [K]</th>
<th>T∗</th>
<th>µ∗</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin ice Dy$_2$Ti$_2$O$_7$</td>
<td>4.33</td>
<td>1/112</td>
<td>1</td>
<td>0.5</td>
<td>4.46</td>
<td>0.163</td>
<td>1.453</td>
</tr>
<tr>
<td>Water ice</td>
<td>2.75</td>
<td>0.58</td>
<td>3.1</td>
<td>253</td>
<td>6670</td>
<td>0.038</td>
<td>1.01</td>
</tr>
<tr>
<td>Pure water</td>
<td>3.01</td>
<td>1</td>
<td>80.1</td>
<td>298</td>
<td>6000</td>
<td>0.44</td>
<td>8.93</td>
</tr>
<tr>
<td>Acetic acid$^3$ (aq) (1 mmol/dm$^3$)</td>
<td>0.98</td>
<td>1</td>
<td>80.1</td>
<td>298</td>
<td>4350</td>
<td>0.140</td>
<td>2.06</td>
</tr>
<tr>
<td>AOT$^4$ in cyclohexane (0.1 mmol/dm$^3$)</td>
<td>25</td>
<td>1</td>
<td>2.02</td>
<td>298</td>
<td>4900</td>
<td>0.090</td>
<td>1.48</td>
</tr>
<tr>
<td>Anhydrous orthophosphoric acid$^5$</td>
<td>2.6</td>
<td>1</td>
<td>61</td>
<td>383</td>
<td>3300</td>
<td>0.36</td>
<td>3.13</td>
</tr>
<tr>
<td>Na-Ca-SiO$_2$ glass$^6$</td>
<td>5</td>
<td>1</td>
<td>8</td>
<td>302</td>
<td>2000</td>
<td>0.07</td>
<td>0.5</td>
</tr>
<tr>
<td>Methemoglobin$^7$ (83 µmol/dm$^3$)</td>
<td>1.6</td>
<td>1</td>
<td>80.1</td>
<td>298</td>
<td>3200</td>
<td>0.23</td>
<td>2.74</td>
</tr>
</tbody>
</table>

In this study, we simulated a system corresponding to Dy$_2$Ti$_2$O$_7$ spin ice parameters between 0.41 K and 0.8 K. We also simulated a lattice electrolyte with parameters \{a = 5 Å, q/e = 1, εr = 20\}, chemical potentials ranging from 2500 K to 2900 K (T∗ = 0.162–0.221), and at temperatures between 270 K and 370 K (|µ∗| = 1.496–1.735). All systems confirm the conclusions presented in the main text.

3. **Supplementary Discussion: Non-equilibrium quantities**

Conceptually, it is important to distinguish between thermodynamic quantities in equilibrium and their extension to the field-driven case. Practically, the only observable out of equilibrium is the concentration quotient \(K_c(E) = n_f(E)^2/(2n_b(E))\). If we relate it to the dissociation constant and the activity coefficient in the same way as in equilibrium, \(K_c(E) = K(E)/\gamma^2(E)\), we face the problem of separating the increase in free charge density – because the field increases the dissociation – from the decrease in free charge density – due to the reduced screening.

The conductivity measurements in strong electrolytes show that the ionic atmosphere cannot establish itself in strong external fields (first Wien effect). It is safe to assume \(\gamma(E) \rightarrow 1\) in high fields. Practically, strong fields in this context corresponds to the length scale ordering \(\ell_E < \ell_D\). In this limit Onsager’s prediction matches the simulations and experiments well. Onsager’s expression for \(K(E)\) is justified and \(\gamma(E)\) goes from \(\gamma(0)\), as predicted by Bjerrum’s theory, to high field limit of unity.

An early discussion of non-equilibrium quantities in the context of the second Wien effect can be found in Patterson and Freitag$^8$ where the “un-screening coefficient” was proposed as a designation for the non-equilibrium activity coefficient.

4. **Supplementary Discussion: Buffering**

Two types of double equilibrium can be distinguished by the source of charges: dissociating molecules or creation from the vacuum. The creation from vacuum can also be modelled as a
“reaction” of two empty sites in case of ice and spin ice. In this section, we present a simple
calculation to explicitly show how the double equilibrium shifts due to the second Wien effect.
The result for dissociating molecules is stated in Onsager’s paper \(^9\) without calculation. Charge
generation exhibits similar properties. Both cases show buffering of the bound charge concentration
in the dilute weak electrolyte limit \(n_e \gg n_f \gg n_b\).

In the following, \(n_m\), \(n_e\), \(n_f\), and \(n_b\) are the densities of molecules, empty sites, free ions, and
bound ions, respectively. Densities after the equilibrium shifts are denoted with a prime, e.g. \(n'_f\).

The double equilibrium \(0 \rightleftharpoons (+) \rightleftharpoons (+) + (-)\) where charges are created from molecules is
described by:

\[
\frac{n_b}{2} = K_m n_m \tag{1}
\]
\[
\left(\frac{n_f}{2}\right)^2 = K_c n_b/2 \tag{2}
\]
\[
2n_m + n_b + n_f = 1, \tag{3}
\]

where we have normalized the total concentration to unity (mole fraction). Substituting the first
two equations into the last gives:

\[
2\left(\frac{n_f}{2}\right)^2 (1 + 1/K_m)/K_c + n_f - 1 = 0, \tag{4}
\]

which is equivalent to the equation for free charge density in case of a single equilibrium \((+-) \rightleftharpoons (+) + (-)\) with dissociation concentration quotient \(K_d = K_c/(1 + 1/K_m)\). The Wien effect gives
for the dissociation constant in field \(K_d(E) = K_d\gamma(0)^2 F(\ell_T/\ell_E)\):

\[
\gamma(0)^2 F(\ell_T/\ell_E)n_f^2/2 + n'_f - 1 = 0. \tag{5}
\]

Expressing \(n'_f\) in terms of the equilibrium concentration gives:

\[
\frac{n'_f}{n_f} = \frac{\gamma(0)^2 F(\ell_T/\ell_E)n_f}{2n_b} \left(1 + \frac{4(1 - n_f)}{\gamma(0)^2 F(\ell_T/\ell_E)n_f^2} - 1\right) \tag{6}
\]

If \(n_f \ll 1\), we can expand into series:

\[
\frac{n'_f}{n_f} = \gamma\sqrt{F(\ell_T/\ell_E)} - \gamma(0)^2 F(\ell_T/\ell_E) + O(n_f^2) = \gamma\sqrt{F(\ell_T/\ell_E)} + O(n_f). \tag{7}
\]

The total concentration of bound pairs and molecules \((1 - n_f) \simeq 1\) stays essentially unchanged as
well as the equilibrium between them. The bound pair concentration is constant to first order:

\[
\frac{n'_b}{n_b} = 1 + O(n_f). \tag{8}
\]
The bound pairs in this case can be considered as precursors of the fully associated molecules. The Wien effect is a non-equilibrium process where the actual rates whose ratio gives the dissociation quotient matter. In addition, the actual rates of transformation between molecules and bound pairs have to be faster than the rates involving exchange between free and bound charges.

The double-equilibrium with charge generation \((00) \rightleftharpoons (+-) \rightleftharpoons (+) + (-)\) studied in the main text can be described by the following set of equations:

\[
\begin{align*}
n_b/2 &= K_e n_e^2 \\
(n_f/2)^2 &= K_c n_b/2 \\
n_e + n_b + n_f &= 1.
\end{align*}
\]

Application of the external field shifts the concentration quotient \(K_c\) to \(K_c(E) = K_c \frac{\gamma(0)^2 F(\ell_T/\ell_E)}{\gamma \sqrt{F(\ell_T/\ell_E)n_f}}\), while the creation constant \(K_e\) stays constant. Solving for the in-field concentrations \(n'_f\) and \(n'_b\) in terms of the zero-field ones, we find:

\[
\begin{align*}
n'_f/n_f &= \frac{\gamma(0)^2 F(\ell_T/\ell_E)n_f}{2n_b} \\
&\times \left[ \sqrt{\left( 1 + \frac{1 - n_b - n_f}{\gamma \sqrt{F(\ell_T/\ell_E)n_f}} \right)^2 + \frac{4n_b}{\gamma(0)^2 F(\ell_T/\ell_E)n_f^2}} - \left( 1 + \frac{1 - n_b - n_f}{\gamma \sqrt{F(\ell_T/\ell_E)n_f}} \right) \right] \\
n'_b/n_b &= \frac{\gamma(0)^2 F(\ell_T/\ell_E)n_f^2}{4n_b^2} \\
&\times \left[ \sqrt{\left( 1 + \frac{1 - n_b - n_f}{\gamma \sqrt{F(\ell_T/\ell_E)n_f}} \right)^2 + \frac{4n_b}{\gamma(0)^2 F(\ell_T/\ell_E)n_f^2}} - \left( 1 + \frac{1 - n_b - n_f}{\gamma \sqrt{F(\ell_T/\ell_E)n_f}} \right) \right]^2.
\end{align*}
\]

Expanding the previous result \((n_e \gg n_f \gg n_b)\) yields:

\[
\begin{align*}
n'_f/n_f &= \gamma \sqrt{F(\ell_T/\ell_E)} + \left( \gamma \sqrt{F(\ell_T/\ell_E)} - \gamma(0)^2 F(\ell_T/\ell_E) \right) n_f + \mathcal{O}(n_b, n_f^2) \\
&= \gamma \sqrt{F(\ell_T/\ell_E)} + \mathcal{O}(n_b, n_f) \\
n'_b/n_b &= 1 + 2 \left( 1 + \gamma \sqrt{F(\ell_T/\ell_E)} \right) n_f + \mathcal{O}(n_b, n_f^2) \\
&= 1 + \mathcal{O}(n_b, n_f)
\end{align*}
\]

The fraction of bound pairs stays constant up to a linear order in \(n_b\), which is of the order \(10^{-5}\) at \(\{\mu^* = -1.45, T^* = 0.140\}\).

To sum up, the requirements for the kinetics presented in the main text to hold are that empty sites dominate (weak electrolyte) and that the charge creation kinetics is faster than the bound pair dissociation. This is indeed the case in MC simulations, where charge creation is a single step
process, while bound pair dissociation takes multiple steps.

5. Supplementary Discussion: Mobility – Metropolis algorithm, Onsager–Fuoss theory

The Metropolis algorithm yields a relative decrease of mobility with field, resulting from the unity probability of stepping in the field direction and probability \(\exp(-E^*/T^*)\) of moving against the field for each proposed step:

\[
\frac{\omega_0(E^*)}{\omega_0(0)} = 1 - \exp(-E^*/T^*),
\]

where \(\omega_0\) is the mobility of an equivalent non-interacting particle. Note, that since the proposed steps are local, it should be possible to rescale the time by the acceptance probability to obtain a close approximation to the physical time\(^\text{10}\). The mobility would still be field-dependent, as predicted e.g. for monopole hopping in spin ice\(^\text{11}\).

The full zero-field mobility \(\omega(0)\) of the Coulomb gas is further reduced because of the ionic atmosphere slowing down the charge. The zero-field correction is well approximated by Fuoss-Onsager conductivity theory\(^\text{12}\) (which fails when the parameter \(\ell_T/\ell_D\) approaches unity, which happens around \(T^* = 0.16\)). Fuoss-Onsager theory for a binary electrolyte takes the following form:

\[
\frac{\omega(0)}{\omega_0(0)} = 1 - h(\ell_T, \ell_D) = \frac{2 - \sqrt{2} \ell_T}{3 \ell_D} - \frac{1}{3} \left( \frac{\ell_T}{\ell_D} \right)^2 \ln \left( \frac{\ell_T}{\ell_D} \right) - \left( \frac{\ell_T}{\ell_D} \right)^2 N \left( \frac{2\ell_T}{a} \right) + \left( \frac{\ell_T}{\ell_D} \right)^2 \gamma(0)^2 K_c,
\]

\[
T_1 \left( \frac{2\ell_T}{a} \right) = \exp \left( -\frac{2\ell_T}{a} \right) \left( 1 + \frac{2\ell_T}{a} + \frac{1}{2} \left( \frac{2\ell_T}{a} \right)^2 \right),
\]

\[
N \left( \frac{2\ell_T}{a} \right) = 1.4985 + \frac{0.2071 T_1(2\ell_T/a) - 0.03066}{1 - T_1(2\ell_T/a)},
\]

\[
K_c \left( \frac{2\ell_T}{a} \right) = \frac{1}{3} \left( \text{Ei} \left( \frac{2\ell_T}{a} \right) - \frac{a}{2\ell_T} \left( 1 + \frac{a}{2\ell_T} \right) \exp \left( \frac{2\ell_T}{a} \right) \right).
\]

To describe the field dependence of this correction, we use Wilson’s calculation of the first Wien effect (for review see Eckstrom and Schmelzer\(^\text{13}\)), which is known to give the field evolution of the term linear in \(\ell_T/\ell_D\) from the previous equation. We assume that the other terms decay in the same or similar fashion; an assumption that appears \textit{a posteriori} to be a good approximation. We obtain

\[
\frac{\omega(E^*)}{\omega_0(E^*)} = 1 - h(\ell_T, \ell_D) g(\ell_D/\ell_E), \text{ where}
\]

\[
g(x) = \frac{3}{(4 - 2\sqrt{2})x^3} \left[ x\sqrt{1 + x^2} - \arctan \left( \frac{x}{\sqrt{1 + x^2}} \right) - \sqrt{2} x + \arctan \left( \sqrt{2} x \right) \right],
\]
and $g(x)$ is scaled to decrease from unity in zero field to zero in infinite field.

The relative change of the total mobility is thus given by:

$$\frac{\omega(E^*)}{\omega(0)} = \frac{1 - h(\ell_T, \ell_D) g(\ell_D/\ell_E) 1 - \exp(-E^*/T^*)}{1 - h(\ell_T, \ell_D)} \frac{E^*/T^*}{E^*/T^*}. \quad (23)$$

Note that in all calculations we only use terms from the relaxation field and exclude terms related to electrophoresis which does not occur because there is no hydrodynamics in our simulations, which have only stochastic dynamics.

6. Supplementary Discussion: Finite-size effects

The CPU time scales with the number of sites $N \propto L^3$, the memory with $N^2 \propto L^6$, since we save the pre-computed Ewald summation for every pair of sites. Saving the potentials allows us to reduce the multiplicative constant of the $L^3$ CPU time dependence. Because the system size is limited, we need to analyse the finite-size effects influencing the simulations results.

Zero-field concentration of charges shows large finite-size effects if $\ell_D > L/2$. The concentration decreases with system size because the volume accessible is too small for a pair of particles to become unbound. For $\mu^* = -1.45$ and $L = 24$, the simulation temperature must stay above $T^* = 0.130$. Figures in the main text are at $T^* \gtrsim 0.140$, which is sufficiently far above the limit because concentration grows exponentially with temperature.

A different kind of finite size effects appears at high fields. The screening cloud becomes elongated and charges diffuse fast in the field direction. Due to periodic boundary conditions, the charges eventually start to interact with a copy of their screening cloud that has wrapped around the system – a partial revival of screening, which decreases $\gamma(E)$, leading to the observed charge concentration exceeding that in thermodynamic limit. We confirmed the process by inspecting the pair correlation functions. For $\mu^* = -1.45$, $L = 24$, and $T^* = 0.140$ these effects set in above $E^* = 0.08$.

In many relevant materials, it would be interesting to study quasi-two-dimensional geometries of samples. In that case, the zero-field concentration of charges can be modified severely by the first of the above-mentioned finite size effects and a significant unit-cell polarization can be induced by the second. For boundary conditions of finite dielectric constant, the Ewald summation must then be treated carefully.\textsuperscript{14}
7. Supplementary Discussion: Approximation of a Real Systems as a Lattice Electrolyte – the Case of Water Ice

How can we approximate a real material system – in all its complexity – by an idealised lattice electrolyte of the type studied in the main text? Renormalisation group theory provides a general theoretical framework and mathematically rigorous formalism by which to answer such a question, but there are few real systems of interest that are sufficiently simple to allow such an approach. In most practical cases, one would require a controlled series of approximations to effect the passage between the realistic microscopic model and the idealised lattice electrolyte. To illustrate how this can be achieved we consider one specific case, – that of water ice. This example has the added bonus that the occurrence of Wien effect of water ice appears to remain an open question, despite many years of research. Hence the outline given here recommends a long term programme for how to solve this vexing problem.

Water ice\textsuperscript{15,16} is an important real substance and also provides a paradigm for systems with macroscopic degeneracy, having lent its name, for example, to the concept of ‘spin ice’ in magnetism\textsuperscript{17}. Highly realistic models of water ice have been investigated by numerical methods\textsuperscript{18}, particularly with regard to surface properties. The bulk properties of water ice are more elusive to simulation as they rely on a small concentration of defects, and the bulk physics of water ice is necessarily based on a number of simplified models of point defects derived from the basic ice model of Bernal–Fowler\textsuperscript{19} and Pauling\textsuperscript{20}, and the non-equilibrium thermodynamic treatment of Jaccard\textsuperscript{21}.

In the crystal structure of ordinary water ice (Ih) the water ions occupy a tetrahedral lattice with hexagonal space symmetry, the hexagonal relative of the diamond lattice considered here. In the ground state one hydrogen ion or proton occupies each line of contact between oxide ions, but is shifted away from the mid-point of the line so that two protons lie close to, and two lie further away from each oxide. This ‘ice rule’ ensures that the crystal structure is composed of covalently bonded water molecules H\textsubscript{2}O connected by hydrogen bonds. However, as recognised by Pauling\textsuperscript{20} the ice rule does not order the proton subsystem, so water ice has a zero point entropy which may be measured in experiment\textsuperscript{22}.

There are four basic types of electrical point defect in the water ice structure\textsuperscript{21}: the ionic defects H\textsubscript{3}O\textsuperscript{+} and OH\textsuperscript{−} and so called D and L defects in which oxide-oxide contacts are occupied by two or zero protons respectively. Without the presence of D/L defects the drift of ionic defects in water ice in response to an applied electric field would form long chains of polarised water molecules in a manner akin to the celebrated ‘Grotthus mechanism’ of conduction. However, the ensuing loss of entropy from the ground state would then provide a thermodynamic force to oppose and eventually extinguish the direct current (such a scenario occurs for magnetic monopoles in spin ice\textsuperscript{23}). The D/L defects play the crucial role of relaxing the polarisation strings and thus allowing the passage of a direct proton current through the boundaries of the sample.

\textit{A priori} there is no reason to expect that transport of ionic defects in water ice should occur by
Supplementary Figure 1: Eigen & al. measurement of the second Wien effect in water ice.\(^{28}\)

quasi-classical hopping diffusion, and indeed there is experimental evidence of proton tunnelling.\(^{24}\) However Chen et al.\(^{25}\) studied a realistic tight binding model of proton dynamics in the ice structure and showed how proton tunnelling in the disordered potential landscape of water ice can lead to effective classical diffusion on mesoscopic scales. This justifies the approximation of proton dynamics in water ice by classical hopping diffusion.

Ryzhkin\(^{26}\) proposed a pseudospin hamiltonian for water ice, which is equivalent to the so-called ‘dipolar spin ice’ hamiltonian for spin ice materials.\(^{17}\) Applying the transformation of this Hamiltonian described in Ref. 27, we can reduce water ice to a lattice electrolyte of Coulombically interacting $\text{H}_3\text{O}^+$ and $\text{OH}^-$ ions, situated on the oxide lattice and connected by polarisation strings (‘Dirac strings’). The effect of D/L defects may then be accounted for by simply neglecting the strings, in which case we arrive at a lattice electrolyte model, equivalent, in thermodynamic terms, to the one studied here (apart from the change in global symmetry from cubic to hexagonal). Putting in realistic numbers we see that water ice may be justifiably approximated by the lattice electrolyte considered here using the parameters $T^* = 0.038$ and $\mu^* = 1.01$ (Fig. 1a, main text).

The second Wien effect for water ice was considered by Onsager and Dupuis\(^2\), who calculated an appropriate dielectric constant to use in Onsager’s formula. The resulting calculation was found to be consistent with measurements of Eigen and colleagues\(^{28}\) (Supplementary Figure 1). However, while the Wien effect could be of great importance for the electrical response of water ice at high electric fields below the dielectric breakdown of ice (for example for ice crystals in thunder clouds) it has barely been considered since. This may be due in part to the relative ambiguity of the Eigen–Onsager result (Supplementary Figure 1), and in part to confusion concerning early conductivity results that were not corrected for surface effects.\(^{16}\)

A more definite theory of the Wien effect on water ice would clearly rely on theoretical estimation of the several factors identified above that were neglected in the original Onsager-Dupuis treatment. Clearly it would then be possible, in a long term programme, to directly re-introduce many of the
neglected details of water ice into a numerical simulation, including the local polarisation, D/L
defects etc., and to examine their effect on the Onsager theory. Water ice thus affords an example
of the controlled passage from realistic microscopic model to idealised lattice electrolyte.

If one accepts that the various approximations are justified then we immediately arrive at a
**new prediction** concerning the Wien effect in water ice: that the Wien effect of the conductivity of
water ice should be modified by the field-dependent mobility of the lattice electrolyte identified in
this paper. As in other cases (see the main text and Supplementary Discussion) this may go some
way to regularising the slightly confused situation concerning the comparison of experiment with
Onsager’s theory in the case of water ice.

**Supplementary Information – References**

   7, 1–48 (1926).

   (1960)


4. Randriamalala, Z., Denat, A., Gosse, J. P., and Gosse, B. Field-Enhanced Dissociation, the
   (1985).


7. R. Bräunig, Y. Gushimana, G. Ilgenfritz. Ionic strength dependence of the electric dissociation

   108(6), 529–534, (1961)


10. Jaubert, L. D. C. and Holdsworth, P. C. W., Signature of magnetic monopole and Dirac string


