Core merging and stratification following giant impact

Maylis Landeau, Peter Olson, Renaud Deguen & Benjamin Hirsh

This document presents additional information on the values of dimensionless parameters in our experiments and after giant impacts (Section 1), the shape of the residual stratification profile after merging (Section 2), mixing scaling rationale and comparison with previous fluid dynamics literature (Section 3), the effect of $Z$ on core mixing and stratification (Section 4), the effect of temperature on core merging and stratification (Section 5), the effect of multiple impacts on core stratification (Section 6), molecular diffusion and convective erosion of a stratified layer in the outer core (Section 7), dynamo processes driven by fountain-collapse after a giant impact (Section 8) and geochemical signature of fountain collapse during core merging on siderophile elements (Section 9).

1 Dimensionless parameter values

Supplementary Table S1 compares dimensionless parameters values in our experiments and after giant impacts.

Although the values of $Bo$ and $Oh$ in our experiments are far from their values after giant impacts (Supplementary Table S1), they are among the highest and smallest, respectively, reported in previous investigations on coalescence (e.g. Chen et al., 2006; Gilet et al., 2007; Blanchette & Bigioni, 2009) or fragmentation with immiscible liquids (e.g. Ichikawa et al., 2010; Baumann et al., 1992; Wacheul et al., 2014; Landeau et al., 2014), and they are extreme enough for the released liquid to produce turbulence during its fall through the upper liquid, forming a self-similar cloud (Landeau et al., 2014; Deguen et al., 2014). To discuss geophysical implications of our experiments, we assume that within this asymptotic, turbulent regime, the mixing rate $V/V_0$ is independent of $Bo$ and $Oh$.

The above assumption is based on the following observations. First, the Reynolds number, which evolves as $\sqrt{Bo/Oh}$, is larger than about 2000 in our experiments.
Figure S1: Experimental scaling giving the volume of the residual stratified layer normalized by the volume of the released liquid as a function of the inverse density ratio $1/P$, for positive values of $P$ and $Z = 2$. The red line is the least squares best-fitting power-law.

According to fluid mechanics experiments on mixing at a stratified interface (e.g. Burridge & Hunt, 2016; Ellison & Turner, 1959), such Reynolds numbers are high enough for viscosity (and thus $Oh$) not to affect the mixing rate. Second, in our experiments with $P > 0$, $Bo$ remains constant (within uncertainties) for $P < 6 \times 10^{-2}$ and decreases rapidly by a factor 4 between $P = 6 \times 10^{-2}$ and $P = 1.1$. This change in the evolution of $Bo$ around $P = 6 \times 10^{-2}$ does not reflect into the evolution of $V/V_0$, as shown in Supplementary Figure 1, suggesting a weak effect of $Bo$ on the amount of mixing.

2 Residual stratification profiles after merging

For released liquids lighter than the lower liquid ($P > 0$), the residual stratification is localized in a layer below the immiscible interface, with unstratified conditions below that (Supplementary Figure 2a). Whereas laminar merging would produce a sharp stratification below a uniform layer, turbulent merging produces a progressively vanishing stratification below the interface (representing the CMB), as shown in Supplementary Figure 2a, consistent with the velocity anomalies detected at the top of Earth’s core (Helffrich & Kaneshima, 2010; Tang et al., 2015).

For released liquids denser than the lower liquid ($P < 0$), stratification extends to the entire depth of the lower layer (Supplementary Figure 2b). A layer with more intense stratification is formed at the base of the lower layer (representing the center of
<table>
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<tr>
<td>Upper layer depth, $Z$</td>
<td>$z_u/R$</td>
<td>1 — 4</td>
<td>1 — 4</td>
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<tr>
<td>Density difference ratio, $P$</td>
<td>$(\rho_l - \rho_r)/(\rho_r - \rho_u)$</td>
<td>$1.7 \times 10^{-3}$ — 1.1 ($P &gt; 0$)</td>
<td>0 — 0.2 ($P &gt; 0$)</td>
</tr>
<tr>
<td>Bond number, $Bo$</td>
<td>$(\rho_r - \rho_u)gR^2/\sigma_r$</td>
<td>190 — 905</td>
<td>$10^{16} — 5 \times 10^{17}$</td>
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<tr>
<td>Ohnesorge number, $Oh$</td>
<td>$\sqrt{\rho_r \nu_r/\sigma_r R}$</td>
<td>0.0028 — 0.0070</td>
<td>$5 \times 10^{-9} — 6 \times 10^{-7}$</td>
</tr>
<tr>
<td>Reynolds number, $Re$</td>
<td>$UR/\nu_u \propto \sqrt{Bo/Oh}$</td>
<td>$\sim 2000 — 10000$</td>
<td>$\sim 10^{14} — 10^{17}$</td>
</tr>
<tr>
<td>$\rho_l/\rho_u$</td>
<td>1.22</td>
<td>1.5 — 2.2</td>
<td></td>
</tr>
<tr>
<td>$\nu_l/\nu_u$</td>
<td>0.82</td>
<td>0.0045 — 3.3</td>
<td></td>
</tr>
</tbody>
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Table S1: Main dimensionless parameters and their typical values in this study and after a giant impact, where $g$ is the acceleration due to gravity, $R$ the equivalent spherical radius of the falling blob, $z_u$ the upper layer depth, $\nu$ the kinematic viscosity, $\rho$ the density, $\sigma$ the interfacial tension with the upper fluid and $U$ the velocity of the falling blob at impact with the upper-lower interface. The subscripts $r$, $u$ and $l$ denote the released, upper and lower fluids, respectively. Estimations of dimensionless parameters for giant impacts are obtained assuming: $\sigma_r \approx 1$ J m$^{-2}$ for the surface tension between liquid metal and liquid silicates (Chung & Cramb, 2000), projectile core sizes ranging from about 1000 km in radius (the smallest projectiles able to melt the entire protomantle depth according to melting scalings from Tonks & Melosh, 1993) to about 3000 km (equal-sized projectiles), an upper bound of 10% for the density difference between the projectile cores (Poirier, 1994; Anderson & Isaak, 2002), a density of liquid metal and liquid silicates at core-mantle boundary depths in the range 9000 — 11000 kg m$^{-3}$ (Morard et al., 2013) and 5000 — 6000 kg m$^{-3}$ (Miller et al., 1991), respectively, a dynamic viscosity for fully-liquid magma oceans in the range of $0.01 — 0.1$ Pa s (Karki & Stixrude, 2010), a dynamic viscosity for liquid metal in the range $10^{-3} — 5 \times 10^{-2}$ Pa s (de Wijs et al., 1998; Funakoshi, 2010), and a value 10 m s$^{-2}$ for $g$. The above projectile sizes lead to $Z$ varying within 2 — 4, and we extend this range to 1 to account for possible excavation during the impact.
Figure S2: Residual stratification after merging. Depth in the lower layer (i.e. downward distance from the immiscible interface separating the upper and lower liquids) as a function of the concentration in released fluid $C$ averaged in the horizontal direction. a, Released liquid lighter than the lower liquid, $P \approx 1.68 \times 10^{-2}$, $Z \approx 2$. b, Released liquid denser than the lower liquid, $P \approx -1.63 \times 10^{-2}$, $Z \approx 2$. Uncertainties on horizontally averaged concentrations are of about 15%.

The core). This layer originates from incomplete mixing between the dense descending plumes and the protocore during the overturn event (Figure 3b).

3 Mixing scaling: Rationale and comparison with existing fluid dynamics literature

In this Section, we provide a rationale for our experimental mixing scaling ($V/V_0 = c_1 P^{c_2} Z^{c_3}$, where $c_1 = 1.13 \pm 0.3$, $c_2 = -0.34 \pm 0.02$ and $c_3 = 0.89 \pm 0.08$) based on comparison with previous studies on turbulent mixing at a density interface. Specifically, we show that the sign of the power-law exponents in our scaling laws is consistent with previous results, providing support for our scaling laws.
Although our experiments are the first to investigate turbulent mixing following impact at an interface between immiscible liquids, the relevant configuration for core merging after giant impacts (Figure 1a), similar processes have been extensively studied in miscible fluids. Turbulent mixing at a miscible density interface plays an important role in the atmosphere and the oceans (discussed in Shrinivas & Hunt, 2014) and, therefore, it has been investigated in various configurations, including the impingement of a turbulent thermal (Zhang & Cotel, 2000; Cotel & Kudo, 2008), a turbulent vortex ring (Linden, 1973; Dahm et al., 1989), a fountain (Lin & Linden, 2005) or a turbulent plume (Baines, 1975; Kumagai, 1984; Cardoso & Woods, 1993) at a density interface.

One important finding of the above previously published studies is that the main parameter controlling the entrainment rate is the ratio between buoyancy forces at the interface (mitigating mixing) and inertia (enhancing mixing), called the Richardson number, and defined by

\[ Ri = \frac{\Delta \rho gl}{\rho U^2}, \]  

(1)

where \( U \) is the characteristic velocity, \( \Delta \rho \) the density difference across the interface, \( \rho \) the density of the surrounding fluid and \( l \) the length scale of the incident turbulence at the interface. Specifically, the above studies find that the relation between the entrainment rate and the Richardson number obeys a power-law given by

\[ \frac{U_e}{U} = \gamma Ri^{-n}, \]  

(2)

where \( U_e \) is the entrainment velocity at the interface, and \( \gamma \) and \( n \) are positive dimensionless constants. Equation (2) is widely accepted and has been verified by numerous experimental studies (e.g. Linden, 1973; Baines, 1975; Kumagai, 1984; Baines et al., 1993; Cardoso & Woods, 1993). However, the value of the exponent \( n \) is debated: values \( n = 1/2 \) (Baines et al., 1993), \( n = 1 \) (Cardoso & Woods, 1993) and \( n = 3/2 \) (Linden, 1973; Baines, 1975; Kumagai, 1984) have been reported, depending on the source flow.

Our experimental results are qualitatively consistent with the above results and rationale. At fixed aspect ratio \( Z \), the density ratio \( P \) can be interpreted as a Richardson number: the density difference between the released and upper liquids (the denominator in \( P \)) is proportional to the inertia acquired during fall in the upper liquid, and the density difference between the released and lower liquids (the numerator in \( P \)) is proportional to the stabilizing buoyancy forces that mitigate mixing during the spreading of the gravity current in the lower liquid (Figure 2a). Therefore, the negative exponent \( c_2 \) in the power-law dependence of the mixing ratio \( V/V_0 \) on \( P \) is qualitatively consistent with equation (2). The positive exponent \( c_3 \) in the dependence on \( Z \) is also qualitatively consistent with equation (2): in our experiments, the velocity of the released volume continuously increases during the fall in the upper layer, implying a negative correlation between \( Z \) and the Richardson number.
For a more quantitative comparison with previous results, the Richardson number in our experiments can be defined by

$$Ri = \frac{(\rho_l - \rho_r)gR}{\rho_l U^2},$$  \hspace{1cm} (3)

where $U$ is the velocity upon impact at the interface between the upper and lower liquids. The velocity has no physical reason to depend on $(\rho_l - \rho_r)$ before entering the lower liquid, therefore we can assume that $U^2 = gR \cdot f_1(Z, (\rho_r - \rho_u)/\rho_u)$, where $f_1$ is a dimensionless parameter independent of the density ratio $P$. The Richardson then becomes

$$Ri = (\rho_l - \rho_r)/\rho_l \cdot f_1,$$

which can be re-written as

$$Ri = P \cdot f_2\left(Z, \frac{\rho_r - \rho_u}{\rho_u}\right),$$  \hspace{1cm} (4)

where $f_2$ is a dimensionless parameter that depends on $Z$ and $(\rho_r - \rho_u)/\rho_u$, but not on $P$.

Determining the form of the parameters $f_1$ and $f_2$ is not straightforward. For small $Z$, the velocity of a turbulent cloud increases with the distance from the source according to the equations of motions under the classical turbulent entrainment assumption (Morton et al., 1956), consistently with the observed trend in our experiments. However, for large distances, the same model predicts that the velocity obeys $U \propto 1/Z$ (e.g. Bush et al., 2003) and this transition in behavior occurs when $Z = O(R/\alpha)$, where $\alpha$ is a constant called entrainment coefficient, equal to 0.23 for immiscible clouds (Landeau et al., 2014). The values of $Z$ in our experiments (and in giant impacts) lie within this transitional zone and, therefore, $U$ does not follow any simple scaling. For this reason, we focus on fixed $Z$ values in the following.

Previous results have been reported in terms of entrainment rate $U_e/U$ whereas, in our experiments, we measure the total amount of mixing (or entrainment) $V/V_0$, more appropriate to discuss implications to Earth’s core structure after giant impacts. For comparison, we assume a constant entrainment rate occuring on an advective time scale, which leads to $V/V_0 \propto U_e/U$. For constant $Z$ values, according to relation (4), our experimental scaling law $V/V_0 \propto P^{-0.34}$ can then be converted into

$$\frac{U_e}{U} \propto Ri^{-0.34}.$$  \hspace{1cm} (5)

The power-law scaling (5) is equivalent to equation (2) with $n = 0.34$. The sign of $n$ is consistent with previous studies but its value is smaller than previously reported values for mixing at a density interface between miscible fluids. Given that the value of $n$ already varies within studies involving miscible fluids (Baines et al., 1993; Cardoso & Woods, 1993; Linden, 1973; Baines, 1975; Kumagai, 1984), it is not surprising that, in our experiments involving immiscible liquids, $n$ takes a value different from previously published values. The small value of $n$ in our experiments likely comes from
Figure S3: Same as Figure 4b but accounting for variations of the aspect ratio Z. We assume a constant core-to-mantle ratio and a magma ocean thickness equal to the sum of the projectile and protoplanet mantle thicknesses.

the presence of immiscible drops of upper liquid in the collapsing fountain (Figure 2a): the entrained drops introduce another Richardson number, which is the relevant parameter at length scales much larger than the drop size, and which corresponds to the ratio between inertia and the buoyancy forces associated with the density difference between the upper and lower liquids. We note that this second Richardson number is on the order of 1 in both our experiments and giant impacts.

4 Effect of $Z$ on core stratification after giant impacts

Supplementary Figure 3 shows the resulting regime diagram for the final state of the protocore as obtained using the scaling law in Figure 4a when accounting for variations in $Z$, assuming that the magma ocean thickness is equal to the sum of the projectile and protoplanet mantle thickness. Supplementary Figure 3 shows that our conclusions for core stratification after giant impacts are weakly affected by accounting for variations in $Z$. Specifically, we find that a projectile with a core-to-protocore mass ratio equal to 0.03 – 0.04 and an initial core density contrast of 5% – 7% produced a stratified layer at the top of Earth’s core of ∼ 200km-300km compatible with that detected by seismology (Helffrich & Kaneshima, 2010).
5 Effect of temperature on core merging and stratification after giant impacts

Shock heating of the projectile core and the protomantle is considerable after a giant impact (Arkani-Hamed & Olson, 2010; Monteux et al., 2013), causing significant temperature variations between the projectile core and the protocore. Temperature variations can therefore play an important role in setting the residual stratification after a giant impact. According to our experiments, the situation of geophysical interest is where the projectile core is enriched in light elements, able to produce a long-lived stratified layer at the top of Earth’s core. In the latter situation, both temperature and composition have stabilizing effects (they are both associated with negative density ratios $P$) and, therefore, the general dynamics, corresponding to the sequence in Figures 2a and 3a, would remain unchanged in the presence of temperature differences. We note that the scaling and the regime diagram in Figure 4 apply to any density difference, irrespective of their origin (temperature or composition).

The temperature profile after a giant impact is poorly constrained (Monteux et al., 2013; Reufer et al., 2012). However, taking typical values of $1000 - 2000$ K for the temperature difference between the protocore and the projectile core (Reese & Solomatov, 2010; Monteux et al., 2013; Melosh, 1990), and a thermal expansion coefficient of $1.3 \times 10^{-5} \text{ K}^{-1}$ for the outer core (Vocadlo et al., 2003), we obtain density differences due to temperature of about $1.5 - 2.5\%$. Injecting such values in our scaling laws, we find that a projectile with a core mass ratio in the range $0.05 - 0.06$ and an initial compositional density contrast of $3.2\% - 3.5\%$ are capable of producing a 300 km layer with a 0.75% compositional density contrast, comparable to the layer inferred from seismology (Tang et al., 2015; Helfrich & Kaneshima, 2010). Thus, accounting for temperature effects does not significantly affect our conclusions on the origin of this stratified layer and on the properties of the moon-forming impact.

6 Effect of multiple impacts on core stratification

Our conclusions concerning the moon-forming impact would be little affected were the stratification produced by multiple impacts; in that case, Figure 4b requires each projectile core, including the moon-forming impactor, be smaller than $0.05 - 0.07$ protocore masses in order to produce the seismically-observed stratification (Helfrich & Kaneshima, 2010), assuming an upper bound of 10% for the core density contrast.
7 Molecular diffusion and convective erosion of a compositionally stratified layer

Taking reasonable values of $10^{-9} - 6 \times 10^{-9}$ m$^2$/s for anion diffusivities $D$ in the outer core (Poirier, 1988; Dobson, 2002), we obtain a diffusion length scale $\sqrt{D\tau}$ equal to $12 - 29$ km, where $\tau$ is the age of the Earth. To be as conservative as possible, we allow for a diffusion length scale from 10 km to 100 km.

To investigate the possible convective erosion of a 300 km stratified layer with a density contrast of 0.75%, we use the results from Levy & Fernando (2002) who investigated the erosion of a linear stratified layer by convection underneath, in a rotating system. Levy & Fernando (2002) find that the erosion rate of the layer $\partial h/\partial t$, where $h$ is the stratified layer thickness, is smaller than $U/Ri$, where $Ri$ is the Richardson number

$$Ri = \frac{N^2 R_c}{U^2}, \quad (6)$$

$U$ the convective velocity, $N$ the buoyancy frequency in the stratified layer, and $R_c$ the thickness of the convective layer. Applied to Earth’s core, the above scaling means that the erosion depth $\delta h$ of the stratified layer over the age of the Earth $\tau$ is smaller than $U^3 \tau/N^2 R_c^2$. Using $\tau = 4.5$ Gyr, the definition of the buoyancy frequency

$$N^2 = \frac{g \Delta \rho}{\rho h}, \quad (7)$$

and $U = 10^{-3}$ m s$^{-1}$ for convective velocities in the core (Bloxham & Jackson, 1991), we find $\delta h \lesssim 50$ m. Note that we use the layer thickness $h$ in equation (7), assuming that the stratification is close to linear within the layer, as supported by our experiments (Supplementary Figure 2a).

Such a small erosion depth indicates that convective erosion is negligible: the above stratified layer, formed after a giant impact, would have persisted through the age of the Earth.

8 Fountain-driven dynamo

Our experiments suggest that fountain collapse (sequence shown in Figure 2) following a giant impact led to the spreading of a turbulent gravity current below the core-mantle boundary. The magnetic Reynolds number (ratio of flow-produced induction to diffusion of magnetic field) associated with this flow likely exceeded $10^8$ – assuming reasonable values for the magnetic diffusivity in the core (de Koker et al., 2012; Pozzo et al., 2012) and buoyancy-driven velocities on the order of $\sqrt{(\rho_l - \rho_u)/\rho_l g R}$. Such values for the magnetic Reynolds number are much larger than the critical value for
dynamo action, typically on the order of $10^3$ for shear flows (Guervilly & Cardin, 2010), implying that fountain collapse could generate an intense global magnetic field if the gravity current circumscribed the entire core.

Previous experiments on the spreading of a turbulent, axisymmetric gravity current (e.g. Hoult, 1972; Hallworth et al., 1996) have shown that the distance $r$ travelled by the front of the current obeys $r \propto (g\Delta \rho/\rho V_f)^{1/4}t^{1/2}$, where $V_f$ is the volume of the collapsing fountain, $\Delta \rho$ the density difference between the fountain and its environment, $\rho$ the density of the surrounding fluid, and $t$ the spreading time. Applying this scaling to giant impacts, using the parameter values given in Supplementary Table 1 and assuming that the projectile core entrained a volume of liquid silicates comparable to its own during the fall in the magma ocean (as observed in our experiments, see main text), we find that the collapsing fountain spread as a gravity current over one core hemisphere in only a few hours. This spreading time is much shorter than the time for transition into a laminar gravity current ($\propto (V_f/\nu g)^{1/3}$ Hoult, 1972, $\sim 500$ days for giant impacts), indicating that the gravity current remained turbulent during its spreading below the core-mantle boundary. We then expect the life-time of the fountain-driven dynamo to be determined by the energy dissipation rate.

A necessary condition for this dynamo to be recorded at the planet surface is that the magma ocean cooled sufficiently fast for surface rocks to solidify and pass below their Curie temperature before the dynamo-generated magnetic field vanished. Although the dynamo was powered by the spreading current during a time of only a few hours, a residual large-scale magnetic field could have been maintained on a much longer time scale fixed by Ohmic decay. The Ohmic decay time of the fundamental mode of the dipole, which is the slowest to decay, is $t_d \propto R_c^2/\eta \pi^2$ (Moffatt, 1978), where $R_c$ is the protocore radius and $\eta$ the core magnetic diffusivity. Using reasonable values for $\eta$ (de Koker et al., 2012; Pozzo et al., 2012), the dipole decay time $t_d$ is on the order of $6 \times 10^4$ years for an Earth-sized planet and $1 \times 10^4$ years for a Mars-sized planet. Meanwhile, previous studies (Solomatov, 2000; Reese & Solomatov, 2006) have shown that a deep, fully-liquid magma ocean solidified in about $t_s \approx 500 - 1000$ years, after which the viscosity of the magma ocean jumped abruptly and a stable crust could form at the surface. Because $t_d \gg t_s$, surface rocks can have been magnetized by fountain-driven dynamos after giant impacts.

The above calculations assume that the gravity current induces a significant large-scale dipole component. Note that the dipole component does not have to be dominant and could represent only a small part of the dynamo field, as long as it is strong enough to be detectable as the planet’s surface. The above assumption may seem optimistic, but is actually reasonable given that previous dynamo simulations in thin shells, less than 10% the depth of the core in thickness, produce a significant, large-scale dipole component (e.g. Stanley et al., 2005).
9 Geochemical signature of core merging

In this section, we provide first-order quantitative arguments showing that the effect of metal-silicate equilibration at core-mantle boundary pressure and temperature during fountain collapse (Figure 2) can be significant, yet consistent with the moderately siderophile elements budget of the Bulk Silicate Earth (BSE).

We denote by $\gamma M$ the mass of the impactor, with $M$ being the final mass of the Earth (i.e. after the moon-forming impact). We assume that the impactor and the Earth have the same core-to-mantle mass ratio, and we note $F \approx 0.32$ the mass fraction of metal. The concentration of element $x$ in the core and mantle are $c_{c\text{imp}}$ and $c_{m\text{imp}}$ in the impactor, and we denote by $c_{m\text{eq}}$ the concentration in Earth’s mantle before the last impact. We denote by $D_{\text{final}}$ the final (estimated) core-to-mantle concentration ratio of the Earth, $D_{\text{imp}}$ the impactor core-to-mantle concentration ratio, and $D$ the thermodynamic metal-to-silicate partition coefficient at the relevant $P, T, f_{O_2}$ conditions.

Denoting by $c_b$ the bulk concentration of element $x$ in the Earth and in the impactor (assumed equal), mass balance allows to write

$$c_{c\text{imp}} = \frac{c_b}{F + \frac{1 - F}{D_{\text{imp}}}}.$$  \hfill (8)

Similarly, the final Earth mantle concentration $c_{m\text{final}}$ can be related to $c_b$ and $D_{\text{final}}$ by

$$c_{m\text{final}} = \frac{c_b}{1 - F + F D_{\text{final}}}.$$  \hfill (9)

Let us now assume that the core of the impactor equilibrates with a mass $\Delta$ times larger of silicates, which has a concentration $c_{m\text{eq}}$ after equilibration. Assuming that the impactor core and the equilibrated silicates reach thermodynamic equilibrium, the concentration in the impactor core after equilibration is $Dc_{m\text{eq}}$. Mass balance then implies that

$$c_{c\text{imp}} + \Delta c_{m\text{eq}} = Dc_{m\text{eq}} + \Delta c_{m\text{eq}} = (D + \Delta)c_{m\text{eq}},$$  \hfill (10)

which gives the concentration in the equilibrated silicates:

$$c_{m\text{eq}} = \frac{c_{c\text{imp}} + \Delta c_{m\text{eq}}}{D + \Delta}.$$  \hfill (11)

The mass of element $x$ added to the mantle by this last impact is equal to the difference of concentration after and before equilibration, $c_{m\text{eq}} - c_{m\text{eq}}$, times the mass of equilibrated silicates, which is $\Delta$ times the mass of the impactor core ($F\gamma M$), i.e.

$$\text{mass of } x \text{ added by the last impact} = (c_{m\text{eq}} - c_{m\text{eq}}) \Delta F\gamma M$$  \hfill (12)

$$= \frac{c_{c\text{imp}} - Dc_{m\text{eq}}}{D + \Delta} \Delta F\gamma M$$  \hfill (13)

$$\leq \frac{c_{c\text{imp}}}{D + \Delta} F\gamma M$$  \hfill (14)
Using equation (8), this gives

\[
\text{mass of } x \text{ added by the last impact} \leq \frac{c_b}{F + \frac{1 - F}{D_{\text{imp}}}} \frac{\Delta}{\Delta + D} F \gamma M \quad (15)
\]

\[
\leq c_b \frac{\Delta}{\Delta + D} \gamma M. \quad (16)
\]

This has to be compared with the total mass of element \( x \) in the BSE, which is equal to the final concentration \( c_m^{\text{final}} \) times the mass of the mantle \((1 - F)M\). Using equation (9), this is

\[
\text{total mass of } x \text{ in BSE} = \frac{c_b}{1 - F + FD_{\text{final}}} (1 - F)M. \quad (17)
\]

Equations (16) and (17) give

\[
\frac{\text{mass of } x \text{ added by the last impact}}{\text{total mass of } x \text{ in BSE}} \leq \left( 1 + \frac{F}{1 - F} D_{\text{final}} \right) \frac{\Delta}{\Delta + D} \gamma. \quad (18)
\]

Let us consider the case of nickel, which has an estimated core-to-mantle concentration ratio \( D_{\text{final}} \simeq 25 \) (Halliday & Woods, 2007). The value of \( D \) at the CMB pressure and temperature is uncertain, but extrapolation of experimental data using the parametrization of Fischer et al. (2015) at \( P = 135 \) GPa, \( T = 6000 \) K, and at an oxygen fugacity of IW-2.3 (i.e. 2.3 log units below the Fe-FeO buffer) gives \( D \simeq 5 \). The value of \( \Delta \) in our experiments is typically around 2. With these values and \( \gamma = 0.1 \), equation (18) predicts that up to about 40% of the mass of Ni in the BSE can be accounted by equilibration at CMB conditions during the last impact. This is clearly significant (but remember this is an upper bound), but not inconsistent with the observed BSE composition since the remaining mass of Ni can be transferred to the mantle at lower \( P \) and \( T \) (corresponding to a more siderophile behaviour of Ni) during the previous impacts. In addition, imperfect equilibration of the impactor core would result in a smaller amount of Ni transferred to the BSE. For example if only half of the impactor core is equilibrated, then the mass of Ni transferred to the mantle would be smaller than 20%.

References


