DOI: 10.1038/NGEO504

Supplementary Methods/Discussion

Quantifying the relationship between the length scales of mantle heterogeneity and their sampling by mid-ocean ridge magmatism, Figure 4 inset (Rubin et al., 2009)

Qualitatively, one expects there to be a reciprocal relationship between length scales of mantle compositional heterogeneity and the sampling density needed to capture it. As the length scale drops, more samples per unit length of ridge will be required to effectively characterise the average mantle composition. But if magma mixing operates during the sampling of these heterogeneities, then some amount of compositional averaging occurs within the magmatic system, and a lower sampling density is required.

These qualitative relationships can be formalised in a simple equation to illustrate the relative scales of heterogeneity size and ridge length as a function of mixing strength, if a number of simplifying assumptions are made. This equation was used to construct the inset of Figure 4 in Rubin et al. (2009)

Firstly, it is assumed that the mantle shows only lateral heterogeneity under the ridge and that each of the mantle portions is evenly sampled by the mantle melting process. These assumptions are unlikely to be correct, but are necessary to illustrate the general relationship between mixing and sampling.

Let the discrete length scale of mantle heterogeneity equal ΔX_i,. Assume that this population of mantle compositions has a normal distribution with standard deviation σ_i for the chemical indicator of interest.

Each spot of basalt on the ridge axis is then assumed to be composed of melts that mix together N_m equal-sized batches of uniform melt from the heterogeneous mantle, and that the full range of mantle compositions may be available for mixing at any one point, implying that the length scale over which mixing can take place is very large compared to ΔX_i. This assumption may also not be correct in detail, but is required for these simple estimates.

In order to estimate the average composition of the mantle under the ridge, N_s samples of basalt are collected. The standard error of estimate of mean mantle composition from the ridge sample

 DOI: 10.1038/NGEO504

is σ_s, which can be calculated from the usual relationship between standard error of a sample and standard deviation of the population:

$$\sigma_s = \frac{\sigma_i}{\sqrt{N_m N_s}}$$

The product of N_m and N_s in the denominator is used to indicate that the averaging takes place in two stages, one corresponding to magmatic mixing and the other to averaging of the MORB sample compositions.

In order to minimise the error on the estimate of the average, more samples are required (high N_s). By inspection, it is clear that the equation provides this behaviour. Likewise, if mixing is very efficient (i.e., N_m is large), it is expected that the error of estimate is small, as captured by the equation. The relative error in an estimate of the average mantle composition is controlled by N_s and N_m, such that if the relative error σ_s / σ_i is constant, the product of N_m and N_s is also constant. If N_s is defined as

$$N_s = L_s M_s$$

where L_s is the ridge-length of the sampling campaign and M_s is the average number of samples per unit length, and

$$L_s = K_s \Delta X_i$$

where K_s is a scaling factor and ΔX_i is the length scale of the mantle heterogeneity, then

$$M_s = \frac{K_l}{N_m \Delta X_i}$$

where K_l is a constant. This simple expression is used to produce the relationships plotted on the inset of Figure 4. In order to estimate the mean of a heterogeneous mantle composition with a given tolerance, then a decrease in heterogeneity scale will require either an increase in sample density, M_s, or an increase in the efficiency of magmatic mixing, which is related to N_m.