Smaller desert dust cooling effect estimated from analysis of dust size and abundance

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Supplementary Figure S1. Contributions of individual particle size ranges to the global dust budget. Size-resolved contributions to (a) the global dust emission rate, (b) global dust mass loading, and (c) global DAOD. Results are obtained by integrating the constraints on size-resolved dust emission, loading, and DAOD (see Fig. 2 and Section 3) between the bin size limits. Clay-sized aerosols ($D \leq 2 \mu m$) make up a small fraction of the emitted dust, but an increasingly large fraction of the mass load and global DAOD. Conversely, very coarse dust ($D \geq 5 \mu m$) accounts for the majority of the emitted dust, but much smaller fractions of the mass load and global DAOD. Error bars denote each particle bin’s CI.
Supplementary Figure S2. Normalized size distribution of emitted dust. The normalized size distribution of dust at emission, as constrained here based on measurements (dashed line with shading for CI; from Fig. 1c), and as assumed in seven AeroCom models (colored lines). Models overestimate the contribution of clay-sized aerosols and underestimate the contribution of very coarse (D ≥ 5 µm) aerosols. Both the experimentally-constrained and modeled size distributions were normalized to unity over the PM$_{2.5}$ size range (see Materials and Methods).

Supplementary Figure S3. The size-resolved direct radiative effect efficiency. Results for the direct radiative effect efficiency, which is the DRE produced per unit global DAOD, are shown for individual particle bins simulated by four different global models in both the SW (a) and LW (b) spectra. The LW radiative effect efficiency includes the effects of LW absorption only; the effect of LW scattering is not included in most global models$^{1,2}$. The horizontal black lines denote the particle bin’s size limits.
Supplementary Figure S4. The size-resolved direct radiative effect (DRE) at top-of-atmosphere (TOA). Panels (a-h) show the DRE calculated for each particle bin of the four global models CESM, GISS, GEOS-Chem, and WRF-Chem. All results were obtained by multiplying the DAOD for each particle bin \([\Delta \tau_d; \text{Eq. } (S.29)]\) by the corresponding model’s radiative effect efficiency \([\Omega(D); \text{Fig. S3}]\); see Section 4 for details. Results in panels (a-d) used the size-resolved DAOD obtained by combining the size-resolved dust loading from the seven AeroCom models with Mie theory for each particle bin (see Section 4.1), whereas panels (e-h) used the constraints on the size-resolved DAOD from this study (see Section 4.2). Panels (i-l) show the difference between the two treatments. Results are shown for DRE due to dust interactions with SW (purple bars), LW (green bars), and all radiation (brown bars). To prevent cluttering the graphs, only the uncertainty in the DRE due to interactions with all radiation is shown. For all four models, correcting the fine size bias of the AeroCom models decreases the cooling by submicron dust \((D < \sim 1 \mu m)\), and increases the warming by coarse dust \((D > \sim 5 \mu m)\).
Supplementary Figure S5. Probability distributions of the dust direct radiative effect (DRE). The probability distribution of the DRE in the SW (a) and LW (b) spectra are shown for each of the three global models, as well as the resulting probability distribution of the total DRE (c). These probability distributions are obtained by combining constraints on the size-resolved global dust optical depth (Fig. 2c) with global model calculations of the DRE per unit optical depth in the SW and LW spectra (Fig. S3).
Supplementary Methods

This section first provides a detailed description of our methodology for obtaining the globally-averaged atmospheric dust size distribution (Section 1), the extinction efficiency (Section 2), the size-resolved global dust emission rate, loading, and optical depth (Section 3), and the dust direct radiative effect (Section 4). We then provide a description of the atmospheric model simulations used in this study (Section 5).

1. Analysis of the globally-averaged atmospheric dust size distribution

Using Equation (3) in the main text, we obtain the globally-averaged particle size distribution (PSD) of atmospheric dust from constraints on the emitted dust PSD (Section 1.1) and the size-resolved dust lifetime (Section 1.2). The resulting globally-averaged atmospheric dust PSD (see Section 1.3) is compared against that obtained by a number of global model simulations (see Section 1.4).

1.1 Globally-averaged dust PSD at emission

The size distribution of dust at emission has been measured by a total of seven studies3-9. These studies measured the emitted dust number size distribution, using optical microscopy of collected dust samples3-5, or optical particle counters used on the ground6-8 or on an airplane flying in the boundary layer9. Below, we describe the procedure for analyzing these measurements, and also provide a description of each study and the methods employed by it. We then describe the procedure for constraining the globally-averaged emitted dust size distribution in Section 1.1.2.

1.1.1 Analysis of the emitted dust PSD data sets

In order to use the seven studies of the emitted dust PSD to constrain the globally-averaged emitted dust PSD, we first need to bring the different data sets on an equal footing. The procedure for this largely follows that described in Ref. 10, with exceptions for each data set described below. Specifically, because emitted dust size distribution measurements are generally well-described by the power law

\[ \frac{dN_{\text{emitted}}}{d \ln D} \propto D^{-2} \]

in the range of 2 – 10 μm (see Fig. 2 in Ref. 10), we fit each set of measurements in that size range for a given wind speed and soil (or location) to this power law. We then normalized measurements at all aerosol sizes for a given soil and wind speed by the proportionality constant in the fitted power law to account for the strong dependence of the dust flux on wind speed and soil type11-13. For a given study, this procedure put measurements at different wind speeds and for different soils on an equal footing, except for the dependence of the shape of the dust PSD on wind speed and soil type11-13.

For each data set, we reduced random errors by averaging over all normalized measurements for a given particle bin for different wind speeds, soils (in the case of Ref. 5), and terrain types (in the case of Ref. 9). This procedure also yielded the standard error of measurements for a given particle bin, which thus does not include any systematic errors inherent in the measurement technique. Since Ref. 3 obtained only one reliable measurement per particle size, the standard error on these single measurements was estimated from the similar measurements of Ref. 4 and 5. The result of the above procedure is plotted in Figure 1c.

We note that the random error obtained through the above procedure is small compared to the spread between measurements from different studies (see, e.g., fig. 5 in Ref. 9 and fig. 3 in Ref. 15). We thus infer that random errors within a data set, which capture measurement uncertainty and the effects of differences in wind speed and soil/terrain type (for Refs. 5 and 9), are small compared to the systematic errors between data sets. This inference is supported by the fact that measurements from different soils within a given study are very small [see Fig. 2 in Ref. 10 and Fig. 5 in Ref. 9], and that changes in wind speed have been shown to have no statistically significant influence on the size distribution of emitted PM10 dust14. This important observation implies that differences in measurements of the emitted dust size distribution are largely due to differences in the measurement technique rather than to differences in the actual size distribution of emitted dust aerosols. Further, since the difference in the emitted dust PSD for different soils, terrain types, and wind speeds is small compared to the systematic error between data sets, we can
consider each data set as an approximate measure of the globally-averaged emitted dust size distribution
(also see discussions in Refs. 10, 14, and 16). This is consistent with the finding that in situ dust size
distributions appear independent of source region16, 17.

1.1.1.1 Gillette data set

The first field measurements of the size-resolved vertical dust flux were made by Gillette and co-
workers3-5. They reported measurements of one sandy loam, two fine sand, and two loamy fine sand soils
in Texas and Nebraska for a range of wind (friction) speeds. These measurements were made using two
single-stage jet impactors at heights of 1.5 and 6 m. The collected aerosols were subsequently analyzed
using optical microscopy to retrieve the size-resolved vertical flux of dust aerosols larger than ~1 μm
geometric diameter. Because systematic errors due to differences in measurement technique between data
sets are much greater than the random errors due to differences in soil and wind speed within a given data
set, we combine the results from the three Gillette studies3-5 into a single data set.

1.1.1.2 Fratini et al. (2009) data set

Fratini et al.6 used eddy covariance to measure the size-resolved flux of dust emitted over a sandy soil
in the Gobi desert in Inner Mongolia, China. The dust particle concentration was measured using an
optical particle counter (OPC), which measured particles with aerodynamic diameters between 0.35 and
9.5 μm. These measurements thus need to be corrected to the geometric size range. The geometric and
aerodynamic diameters are related by18, 19

$$D_d = \sqrt{\frac{\chi \rho_p D_{ac}}{\rho}}$$

(S.1)

where ρp ≈ 2.5 ± 0.2 × 10³ kg/m³ is the density of dust aerosols (see main text), and χ is the dynamic
shape factor, which is defined as the ratio of the drag force experienced by the irregular particle to the
drag force experienced by a spherical particle with diameter Dd18. Measurements of the dynamic shape
factor for mineral dust particles with a geometric diameter of ~10 μm find χ ≈ 1.4 ± 0.120-22. Inserting
this into equation (S.1) then yields that Dd ≈ (0.75 ± 0.04) Dac, where the standard error was obtained
using error propagation23. Note that the Fratini et al. results from the coarser particle bins (> ~ 5 μm) are
unreliable because the efficiency of the air inlet system was not tested and could have produced an under-
sampling of larger particles (Fratini, 2012, personal communication). Consequently, we did not use these
measurements, and normalized the Fratini et al. data over the range of 2–4 μm (instead of 2–10 μm).
Furthermore, because the scatter in the measurements of Fratini et al.6 is substantially greater than that in
the other data sets, we averaged adjacent pairs of particle bins to reduce this scatter.

1.1.1.3 Sow et al. (2009) data set

Sow et al.7 used two optical particle counters at heights of 2.1 and 6.5 m to measure the size-resolved
vertical flux of dust aerosols larger than 0.3 μm. They simultaneously measured the wind speed at several
heights, which they used to obtain the dust flux through the gradient method of Gillette et al.3. Sow et al.7
reported measurements made during three dust storms in Niger for which the average wind friction speed
varied between 0.4 and 0.6 m²; they did not report the soil type.

1.1.1.4 Shao et al. (2011) data set

Shao, Ishizuka, and co-authors8, 24 reported measurements of the vertical dust flux generated by a
strong erosion event during the Japanese Australian Dust Experiment (JADE). The JADE field campaign
took place in 2006 on a flat, fallow agricultural field with a loamy sand soil in southeastern Australia24, 25.
The investigators used optical particle counters at 1.0, 2.0, and 3.5 m heights to measure the particle
concentration in the 0.3–8.4 μm geometric diameter size range (Ishizuka, 2012, personal communication).
Simultaneous wind speed measurements were made with anemometers at 0.50 and 2.16 m height. These
measurements were combined to calculate the vertical dust flux as a function of friction velocity using the
gradient method3, with an added correction for the gravitational settling of dust particles. The measured
wind friction speed was in the range of $<0.20 \text{ m/s}$ to $0.55 \text{ m/s}$. The authors questioned the reliability of the $0.3 - 0.6 \mu m$ size bin (p. 13 of Ref. 8), which is thus not used here.

### 1.1.1.5 Rosenberg et al. (2014) data set

In contrast to the previous data sets, which were all obtained on the ground during active dust emission, the measurements of Rosenberg et al.$^9$ were made from an airplane flying over dusty regions in the central Sahara. The authors obtained measurements of the size-resolved aerosol fluxes up to $300 \mu m$ diameter for four different regions and at three different ranges of the vertical turbulent kinetic energy. Rosenberg et al. obtained these size-resolved aerosol fluxes using eddy covariance, which was facilitated by high frequency measurements of the size-resolved aerosol concentration (using several different OPCs) and the 3D wind (using pitot probes). These flux measurements were made in the lower portion of the atmospheric boundary layer, at altitudes ranging between $\sim 100 – 1000 \text{ m}$. We only use the size-resolved aerosol flux measurements with particle sizes $\geq 0.5 \mu m$ because (i) measurements of the SAMUM campaign over the Sahara desert$^{26-28}$ showed that aerosols with diameter $\leq 0.5 \mu m$ are largely not dust aerosols$^{29}$, and (ii) the fraction of aerosols $\leq 0.5 \mu m$ that is dust is often coated in volatiles$^{26}$, which was not accounted for in Rosenberg et al.$^9$. Conversely, we assume that the aerosol fluxes $> 0.5 \mu m$ are entirely due to dust$^{26-28}$.

### 1.1.2 Obtaining the globally-averaged emitted dust PSD

After the previous section described the analysis of the emitted dust PSD data sets, we now describe the procedure that uses these data sets to determine the most likely globally-averaged emitted dust PSD and its 95\% confidence interval. We obtain the most likely globally-averaged emitted dust size distribution using a statistical model that accounts for systematic errors inherent in each study’s measurement methodology, which allows us to better constrain the emitted dust size distribution than otherwise possible. Specifically, we (i) fit each emitted dust PSD data set to an analytical function, (ii) use these analytical functions in a maximum likelihood procedure that explicitly consider the systematic errors between data sets, and (iii) use a bootstrap procedure$^{30,31}$ to obtain the 95\% confidence interval.

#### 1.1.2.1 Fitting the emitted dust PSD

We thus first fit each data set to the analytical expression of the emitted dust PSD obtained from brittle fragmentation theory$^{10}$, which is in good agreement with each data set$^{9,15}$. This expression is given by

$$
\frac{dV_{\text{emit}}}{d \ln D} = \frac{D}{c_V} \left[ 1 + \text{erf} \left( \frac{\ln(D/D_M)}{\sqrt{2} \ln \Omega_s} \right) \right] \exp \left( - \left( \frac{D}{\lambda} \right)^3 \right),
$$

(S.2)

where $V_{\text{emit}}$ is the normalized volume concentration of emitted dust aerosols with geometric size $D$, $c_V$ is a normalization constant, and $\Omega_s$, $D_M$, and $\lambda$ are model parameters whose significance are discussed in Ref. $^{10}$. We used a non-linear least-squares analysis$^{23}$ to fit equation (S.2) to each data set, which yielded the least-squares estimates of the model parameters ($\Omega_s$, $D_M$, and $\lambda$), their errors, and their covariances. Because systematic errors between data sets are much larger than the random errors within each data set (see discussion above), we assumed that the relative error, which is due to both random and systematic errors, is equal for all data points within a data set.

#### 1.1.2.2 Using the maximum likelihood method to estimate the most likely globally-averaged emitted dust PSD

The above procedure thus yields five values of $\Omega_s$, $D_M$, and $\lambda$, with their errors and covariances. We use these five estimates in a likelihood procedure to obtain the maximum likelihood estimates (MLEs) of the globally-representative values, $\mu_{\Omega_s}$, $\mu_{D_M}$, and $\mu_{\lambda}$, that describe the globally-averaged emitted dust PSD per equation (S.2). This procedure explicitly accounts for the systematic error affecting the five fitted values of $\Omega_s$, $D_M$, and $\lambda$. That is, as we describe in more detail below, we assume that each data set’s values of $\Omega_s$, $D_M$, and $\lambda$ are drawn from normal distributions for which the standard deviation represents
the average systematic errors \( \tau_{\Omega_s}, \tau_{D_M}, \) and \( \tau_\lambda \) between data sets. We then use maximum likelihood procedures to find both the globally-representative parameter values \((\bar{\mu}_{\Omega_s}, \bar{\mu}_{D_M}, \) and \( \bar{\mu}_\lambda)\) and the characteristic systematic error between data sets \((\tau_{\Omega_s}, \tau_{D_M}, \text{ and } \tau_\lambda)\), thereby propagating these errors into the uncertainty on the emitted dust size distribution (shaded region in Fig. 1c).

**Procedure for obtaining \( \bar{\mu}_{\Omega_s} \) and \( \bar{\mu}_{D_M} \).** Since \( \Omega_s \) and \( D_M \) occur jointly inside the error function in equation (S.2), their values are correlated. Making the standard assumption of normally-distributed errors, we describe the likelihood of obtaining the fitted parameters \( \Omega_{s,i} \) and \( D_{M,i} \) with a bivariate normal distribution that is centered around the ‘true’ values \( \mu_{\Omega_{s,i}} \) and \( \mu_{D_{M,i}} \) for the particular data set \( i \). That is, as the standard errors \( \sigma_{\Omega_{s,i}} \) and \( \sigma_{D_{M,i}} \) approach zero and equation (S.2) becomes a perfect to the measured data, \( \Omega_{s,i} \) and \( D_{M,i} \) respectively approach \( \mu_{\Omega_{s,i}} \) and \( \mu_{D_{M,i}} \).

The joint probability of \( \Omega_{s,i} \) and \( D_{M,i} \) is thus given by

\[
\begin{bmatrix}
\Omega_{s,i} \\
D_{M,i}
\end{bmatrix} \sim N_2 \left( \begin{bmatrix} \mu_{\Omega_{s,i}} \\ \mu_{D_{M,i}} \end{bmatrix}, \Sigma_i \right),
\]

where \( \Sigma_i = \begin{pmatrix} \sigma^2_{\Omega_{s,i}} & \rho_i \sigma_{\Omega_{s,i}} \sigma_{D_{M,i}} \\ \rho_i \sigma_{\Omega_{s,i}} \sigma_{D_{M,i}} & \sigma^2_{D_{M,i}} \end{pmatrix} \) (S.3)

where \( N_2 \) denotes a bivariate normal distribution, for which the first term in parentheses denotes the distribution’s median, and the second term (\( \Sigma_i \)) the covariance matrix. Furthermore, \( \rho \) is the correlation between \( \Omega_{s,i} \) and \( D_{M,i} \) for the particular data set \( i \); \( \sigma_{\Omega_{s,i}} \) and \( \sigma_{D_{M,i}} \) are obtained from the least-squares fitting procedure of equation (S.2) to data set \( i \).

Even if all random error is eliminated from the measurements, systematic errors would cause the values of \( \mu_{\Omega_{s,i}} \) and \( \mu_{D_{M,i}} \) obtained for each of the five data sets to be offset from the ‘true’ \( \bar{\mu}_{\Omega_s} \) and \( \bar{\mu}_{D_M} \) that actually occur in the real world. Since differences in soil properties and wind speed seem to have only limited impact on the emitted dust PSD (see discussion above), we infer that these systematic errors are largely due to differences in experimental techniques\(^{19}\). We assume that the systematic errors for the five data sets are drawn from a bivariate normal distribution centered around zero, with unknown variances \( \tau^2_{\Omega_s} \) and \( \tau^2_{D_M} \), respectively, for \( \Omega_s \) and \( D_M \). That is,

\[
\begin{bmatrix}
\bar{\mu}_{\Omega_s} \\
\bar{\mu}_{D_M}
\end{bmatrix} \sim N_2 \left( \begin{bmatrix} \bar{\mu}_{\Omega_s} \\ \bar{\mu}_{D_M} \end{bmatrix}, \Gamma \right),
\]

where

\[
\Gamma = \begin{pmatrix} \tau^2_{\Omega_s} & \eta \tau_{\Omega_s} \tau_{D_M} \\ \eta \tau_{\Omega_s} \tau_{D_M} & \tau^2_{D_M} \end{pmatrix},
\]

(S.6)

where \( \eta \) is the correlation between \( \bar{\mu}_{\Omega_s} \) and \( \bar{\mu}_{D_M} \), which we estimate as the correlation between the 5 values of \( \Omega_{s,i} \) and \( D_{M,i} \), yielding \( \eta = 0.68 \). Combining equations (S.5) and (S.6) then yields

\[
\begin{bmatrix}
\Omega_{s,i} \\
D_{M,i}
\end{bmatrix} \sim N_2 \left( \begin{bmatrix} \bar{\mu}_{\Omega_s} \\ \bar{\mu}_{D_M} \end{bmatrix}, \Sigma_i + \Gamma \right)
\]

(S.7)

The likelihood of obtaining any particular set of \( \Omega_{s,i} \) and \( D_{M,i} \) values from the \( n = 5 \) data sets is then

\[
L = \prod_{i=1}^n \prod_2 \left[ \frac{1}{2\pi} \frac{1}{\sqrt{\sigma^2_{\Omega_{s,i}} + \tau^2_{\Omega_s}}} \exp \left[ -\frac{1}{2(\sigma^2_{\Omega_{s,i}} + \tau^2_{\Omega_s})} \right] \right] = \prod_{i=1}^n \prod_2 \left[ \frac{1}{2\pi} \frac{1}{\sqrt{\sigma^2_{\Omega_{s,i}} + \tau^2_{\Omega_s}}} \exp \left[ -\frac{1}{2(\sigma^2_{\Omega_{s,i}} + \tau^2_{\Omega_s})} \right] \right],
\]

(S.8)

where

\[
Q_i(\Omega_{s,i}, D_{M,i}) = \frac{(\Omega_{s,i} - \bar{\mu}_{\Omega_s})^2 + (D_{M,i} - \bar{\mu}_{D_M})^2}{\sigma^2_{\Omega_{s,i}} + \tau^2_{\Omega_s} + \sigma^2_{D_{M,i}} + \tau^2_{D_M}} - 2r_i \frac{(\Omega_{s,i} - \bar{\mu}_{\Omega_s})(D_{M,i} - \bar{\mu}_{D_M})}{\sqrt{\sigma^2_{\Omega_{s,i}} + \tau^2_{\Omega_s} \sigma^2_{D_{M,i}} + \tau^2_{D_M}}},
\]

(S.9)

and
By maximizing the likelihood function \( L \) with the calculated values of \( \Omega_i \) and \( D_{M,i} \), we estimated the unknown parameters \( \tilde{\mu}_{\Omega_i} = 2.10, \tilde{\mu}_{DM} = 1.52 \mu m, \tau_{\Omega_i} = 0.25, \) and \( \tau_{DM} = 0.54 \mu m \).

**Procedure for obtaining \( \tilde{\mu}_{\lambda} \).** We now describe the procedure for obtaining the MLE of \( \tilde{\mu}_{\lambda} \), which is similar to that for \( \tilde{\mu}_{\Omega_i} \) and \( \tilde{\mu}_{DM} \) described above. \( \tilde{\mu}_{\lambda} \) is the most likely globally-representative value of the parameter \( \lambda \), which affects the shape of the size distribution curve for particle diameters \( \geq \sim 10 \mu m \). Since \( \Omega_i \) and \( D_{M} \) predominantly describe the size distribution for smaller particle sizes, \( \lambda \) is only weakly correlated to \( \Omega \) and \( D_{M} \). For simplicity, we thus consider the fitted parameter \( \lambda_i \) to be an independent parameter that is normally distributed (denoted by \( N \)) around the ‘true’ value \( \mu_{\lambda,i} \) that exists for each particular data set \( i \). The likelihood of obtaining the fitted parameter \( \lambda_i \) is then

\[
\lambda_i \sim N(\mu_{\lambda,i}, \tau_{\lambda}^2).
\]

Combining equations (S.11) and (S.12) then yields the likelihood of obtaining the particular set of \( \lambda_i \) values from the \( n = 5 \) data sets:

\[
L = \prod_{i=1}^{n} N(\mu_{\lambda,i}, \sigma_{\lambda,i}^2 + \tau_{\lambda}^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi(\sigma_{\lambda,i}^2 + \tau_{\lambda}^2)}} \exp \left[ -\frac{(\lambda_i - \mu_{\lambda,i})^2}{2(\sigma_{\lambda,i}^2 + \tau_{\lambda}^2)} \right]
\]

By again maximizing the likelihood function \( L \), we estimated the unknown parameters \( \tilde{\mu}_{\lambda} = 20.5 \mu m \) and \( \tau_{\lambda} = 7.9 \mu m \).

After calculating \( \tilde{\mu}_{\Omega_i}, \tilde{\mu}_{DM}, \) and \( \tilde{\mu}_{\lambda} \) through the above procedures, we used these values to calculate the normalization factor \( c_V \) by forcing the integral over equation (S.2), from \( D = 0.2 \) to \( 20 \mu m \), to unity. The resulting most likely globally-averaged emitted dust size distribution is plotted in Figure 1c.

**1.1.2.3 Obtaining the error in the globally-averaged emitted dust size distribution**

In addition to obtaining the most likely globally-averaged emitted dust size distribution, we also require its 95% confidence interval. We obtained this using a modified bootstrap procedure30, 31:

1. We randomly choose one data set from the total of five emitted dust PSD data sets, and repeat this five times with resampling. This results in a set of five randomly-selected data sets, in which each data set can be represented more than once, or not at all. Note that, since the bootstrapping method requires identical and independently distributed measurements30, 31, we thus necessarily assume that all data sets are independent. This is another reason for combining the three Gillette studies into a single data set.
2. We obtain the values of \( \tilde{\mu}_{\Omega_i}, \tilde{\mu}_{DM}, \) and \( \tilde{\mu}_{\lambda} \) from the procedure described in Section 1.1.2.2.
3. We repeat steps 1 and 2 a large number of times, yielding a large number of possible curves for the globally-averaged emitted dust PSD.
4. For each value of the particle diameter \( D \), the 95% confidence interval is the interval within which 95% of the curves obtained in step 3 lie30, 31. This confidence interval is plotted as gray shading on Figure 1c.

**1.2 Analysis of the globally-averaged and size-resolved dust lifetime**
We constrain the globally averaged size-resolved dust lifetime from the lifetime simulated with nine different climate and chemical transport models. These include GISS (see figure S12 in Ref. 32), GMOD (see table 2 in Ref. 33), CESM (calculated from simulations reported in Ref. 34), MOZART (see Table 2 in Ref. 35), UMI (see table 3 in 36; our Fig.2 shows the geometric mean of the three reported simulations with different meteorological data sets), MERRAero (calculated from simulations accessible at http://opendap.nccs.nasa.gov/dods/GEOS-5/MERRAero37, 38-40), WRF-Chem (see Section 5.1), GEOS-Chem (Section 5.2), and HadGEM (Section 5.3). We use the results of these nine global transport models to constrain the size-resolved dust lifetime in a manner similar to that described above for the emitted dust PSD. That is, first we fit each model result with an analytical function (Section 1.2.1), after which we obtain the most likely globally-averaged size-resolved dust lifetime using the maximum likelihood method (Section 1.2.2), and finally we obtain the 95% confidence interval using the bootstrap method (Section 1.2.3).

### 1.2.1 Fitting the size-resolved dust lifetime

The nine simulation results indicate that the dust lifetime decreases approximately exponentially with particle size (see Fig. 1d):

\[
T(D) = T_0 \exp\left(-\frac{D}{D_{\text{dep}}} \right)
\]  

(S.14)

where \(T_0\) is the lifetime of dust with vanishingly small diameter, which is of the order of 10 days (Fig. 1d), and the constant \(D_{\text{dep}}\) scales the exponential decay of the dust lifetime with particle size. Equation (S.14) can be written as

\[
\ln(T) = a + bD
\]  

(S.15)

where \(a = \ln(T_0)\) and \(b = -1/D_{\text{dep}}\). For a given model \(i\), we used a linear least-squares procedure to fit equation (S.15) to the model results. This yielded the intercept \(a_i\) and slope \(b_i\), the correlation \(\rho\) between intercept and slope, and the errors in the intercept \((\sigma_{a,i})\) and slope \((\sigma_{b,i})\) relative to the ‘true’ values of the intercept \((\mu_{a,i})\) and slope \((\mu_{b,i})\) for the particular climate model \(i\). These errors are caused by internal model error, the finite extent of particle bins, the fact that equation (S.14) is a theoretical and not exact description of the lifetime dependence on particle size, and other sources of error.

### 1.2.2 Using the maximum likelihood method to obtain the most likely globally-averaged dust lifetime

We used these nine estimates of \(a_i\) and \(b_i\) in a maximum likelihood procedure to obtain the most likely globally-representative values of \(\bar{\mu}_{T_0}\) and \(\bar{\mu}_{D_{\text{dep}}}\), which describe the globally-averaged dust lifetime per equation (S.15). Because the slope and intercept of a least-squares fit are correlated\(^2\), we again describe the joint probability of obtaining the intercept \(a_i\) and slope \(b_i\) in equation (S.15) using a bivariate normal distribution, which is centered around the ‘true’ values \(\mu_{a,i}\) and \(\mu_{b,i}\):

\[
\begin{pmatrix} a_i \\ b_i \end{pmatrix} \sim N_2 \left[ \begin{pmatrix} \mu_{a,i} \\ \mu_{b,i} \end{pmatrix}, \Sigma_i \right]
\]

where

\[
\Sigma_i = \begin{pmatrix} \sigma_{a,i}^2 & \rho_i \sigma_{a,i} \sigma_{b,i} \\ \rho_i \sigma_{a,i} \sigma_{b,i} & \sigma_{b,i}^2 \end{pmatrix}
\]

(S.17)

The values of \(\mu_{a,i}\) and \(\mu_{b,i}\) for each climate model are affected by systematic biases, for instance due to errors in the deposition scheme\(^4\), that offset them from the ‘true’ (unbiased) intercept \(\bar{\mu}_a\) and slope \(\bar{\mu}_b\) that actually occur in the real world. Similar to our procedure for constraining the emitted dust PSD (Section 1.1.2), we assume that these systematic errors are drawn from a bivariate normal distribution centered around zero, and with unknown variances \(\tau_a^2\) and \(\tau_b^2\), respectively, for \(a\) and \(b\). That is,

\[
\begin{pmatrix} \mu_{a,i} \\ \mu_{b,i} \end{pmatrix} \sim N_2 \left[ \begin{pmatrix} \bar{\mu}_a \\ \bar{\mu}_b \end{pmatrix}, \Gamma \right],
\]

where

\[
\Gamma = \begin{pmatrix} \tau_a^2 & \eta \tau_a \tau_b \\ \eta \tau_a \tau_b & \tau_b^2 \end{pmatrix}
\]

(S.19)
where \( \eta = -0.75 \) is the correlation between \( \mu_{a,i} \) and \( \mu_{b,i} \). Combining equations (S.18) and (S.19) then yields the joint probability distribution of \( a_i \) and \( b_i \) in terms of their ‘true’ globally-representative values (\( \bar{\mu}_a \) and \( \bar{\mu}_b \)), their mean variance (\( \tau_a^2 \) and \( \tau_b^2 \)), and the standard errors (\( \sigma_{a,i}^2 \) and \( \sigma_{b,i}^2 \)) and covariances (\( \rho_{ab} \)) obtained from the least-squares fitting procedure of equation (S.15) to data set \( i \):

\[
\left( a_i \right) \sim N_2 \left( \left[ \bar{\mu}_a \bar{\mu}_b \right], \Sigma_i + \Gamma \right) = N_2 \left( \left( \bar{\mu}_a \bar{\mu}_b \right), \begin{pmatrix} \sigma_{a,i}^2 + \tau_a^2 & \rho_{a,i} \sigma_{a,i} \sigma_{b,i} + \eta \tau_a \tau_b \\ \rho_{a,i} \sigma_{a,i} \sigma_{b,i} + \eta \tau_a \tau_b & \sigma_{b,i}^2 + \tau_b^2 \end{pmatrix} \right). \tag{S.20}
\]

The likelihood of obtaining the particular set of intercepts and slopes from the least-squares fitting procedure of equation (S.15) to data set \( i \) is then

\[
L = \prod_{i=1}^{n} N_2 \left( \left[ \bar{\mu}_a \bar{\mu}_b \right], \Sigma_i + \Gamma \right) = \prod_{i=1}^{n} \frac{1}{2\pi \sqrt{\sigma_{a,i}^2 + \tau_a^2 \sigma_{b,i}^2 + \tau_b^2}} \exp \left( -\frac{1}{2 \left( 1 - r_i^2 \right)} Q(a_i, b_i) \right), \tag{S.21}
\]

where

\[
Q(a_i, b_i) = \frac{(a_i - \bar{\mu}_a)^2}{\sigma_{a,i}^2 + \tau_a^2} + \frac{(b_i - \bar{\mu}_b)^2}{\sigma_{b,i}^2 + \tau_b^2} - 2r_i \frac{a_i - \bar{\mu}_a}{\sqrt{\sigma_{a,i}^2 + \tau_a^2}} \frac{b_i - \bar{\mu}_b}{\sqrt{\sigma_{b,i}^2 + \tau_b^2}}, \tag{S.22}
\]

and

\[
r_i = \frac{\rho_{a,i} \sigma_{a,i} \sigma_{b,i} + \eta \tau_a \tau_b}{\sqrt{\sigma_{a,i}^2 + \tau_a^2} \sqrt{\sigma_{b,i}^2 + \tau_b^2}}. \tag{S.23}
\]

We obtained the parameters \( \bar{\mu}_a = 2.5, \bar{\mu}_b = -0.22, \tau_a = 0.41 \), and \( \tau_b = 0.03 \) by maximizing the likelihood function \( L \), which yield \( \bar{\mu}_T = 12.5 \text{ days}, \bar{\mu}_{D,\text{dep}} = 4.6 \text{ days}, \tau_T = 5.1 \text{ days}, \) and \( \tau_{D,\text{dep}} = 0.7 \text{ days} \). The resulting most likely globally-averaged dust lifetime is plotted on Figure 1d.

1.2.3 Obtaining the error in the globally-averaged dust lifetime

In addition to obtaining the most likely size-resolved dust lifetime, we also require its uncertainty. We obtained this using a modified bootstrap procedure \(^{30, 31}\) similar to what we used for the emitted dust PSD (Section 1.1.2.3):

1. We randomly chose one data set from the total of nine dust lifetime data sets, and repeat this nine times with resampling. This resulted in a set of nine randomly-selected data sets, in which each data set can be represented more than once, or not at all.
2. We obtained the values of \( \bar{\mu}_a \) and \( \bar{\mu}_b \) from the procedure described above.
3. We repeated steps 1 and 2 a large number (i.e., \( 10^5 \)) of times, yielding a large number of possible curves for the globally-averaged dust lifetime.
4. For each value of the particle diameter \( D \), the 95% confidence interval is the interval within which 95% of the curves obtained in step 2 lie\(^{30, 31}\). This confidence interval is plotted as gray shading on Figure 1d.

1.3 Obtaining the globally-averaged dust size distribution and its uncertainty

With both the MLEs of the globally-averaged emitted dust PSD (Section 1.1) and dust lifetime (Section 1.2) known, we inserted these results into equation (3) to obtain the most likely globally-averaged normalized atmospheric dust PSD (dashed line in Fig. 2a). Furthermore, the bootstrap procedures in Sections 1.1 and 1.2 yield a large number of possible curves for the emitted dust PSD and the size-resolved dust lifetime. Using equation (3), we use these to similarly generate a large number (i.e., \( 10^5 \)) of curves for the size-resolved globally-averaged dust size distribution. The plotted 95% confidence interval (gray shading) is the interval within which 95% of the ensemble values lie\(^{30, 31}\). We find that the volume size distribution of atmospheric dust peaks around 5 \( \mu \text{m} \) (Fig. 2a), which is slightly coarser than a compilation of ground-based measurements\(^{16}\).

1.4 Analysis of atmospheric size distribution in models
Figure 2 shows the most likely atmospheric dust size distribution and its uncertainty. For comparison, we also show the atmospheric dust size distribution reported by seven climate and chemical transport models that participated in the Aerosol Comparison between Observations and Models (AeroCom) project. Specifically, the models included in Fig. 2 are: CAM (see Table 2 in Ref. 44), the GISS ModelE (Figure 2 in Ref. 2), GOCART (see table 3 in Ref. 45), MATCH (see table 8 in Ref. 46), MOZART (see Table 2 in Ref. 35), UMI (see table 3 in Ref. 36; our Fig. 2 shows the geometric mean of the three reported simulations with different meteorological data sets), and LOA (see tables 2 and 3 in Ref. 47). We were unable to locate literature reporting atmospheric dust size distribution for other AeroCom models (SPRINTARS48-50; ECMFW51, UIO_CTM52, 53; LSCE54; ECHAM5-HAM55; MIRAGE56; TM557, 58).

2. Analysis of the shortwave extinction efficiency

We seek to obtain the dust optical depth per unit dust loading that is produced by the globally-averaged atmospheric size distribution (Section 1). To do so, we require $Q_{\text{ext}}(D)$, the globally-averaged extinction efficiency of dust as a function of its particle size, at the wavelength of 550 nm for which the global DAOD is constrained. The extinction efficiency depends on the size, shape, and refractive index of dust aerosols. We thus used measurements to constrain the globally-averaged dust index of refraction (Section 2.1) and dust particle shape (Section 2.2), which we then converted to $Q_{\text{ext}}(D)$ using the single-scattering database of Meng et al. (Section 2.3).

2.1 Globally-averaged dust index of refraction

Measurements of the real refractive index of dust at 550 nm from a variety of source regions and at a range of transport stages (i.e., fresh versus aged dust) are in the approximate range of $m = 1.45 - 1.60$, such that we take the globally-averaged real index of refraction as $n = 1.53 \pm 0.03$. This value is thus intended to account for changes in $n$ during transport due to chemical processing, which might be important for Asian dust, but is likely less important for African dust. Note that, by characterizing the global ensemble of dust aerosols with a single average value for the index of refraction, rather than with a distribution as we do with the shape (see below), we are neglecting any non-linearities in the extinction efficiency with refractive index. Because the dependence of extinction on the real refractive index in the relatively narrow range of 1.45-1.60 is largely linear, the error from this simplification is small compared to other errors in our analysis. Indeed, a sensitivity study indicates that the error associated with this simplification is less than a percent.

Whereas the real refractive index of dust is thus relatively well-known, the imaginary refractive index reported by many in situ studies is substantially larger than that derived from (ground-based or satellite) remote sensing observations. In order to encompass both, we take $10\log(-k) = -2.5 \pm 0.3$. However, since even large variability in the imaginary refractive index has a limited effect on the dust extinction efficiency, this large uncertainty in the dust absorption properties does not contribute substantially to the uncertainty in the global dust loading.

2.2 Globally-averaged distribution of dust particle shapes

Since the uncertainty in dust optical properties produces only limited uncertainty in the extinction efficiency, the main uncertainty in $Q_{\text{ext}}(D)$ arises from the irregular shape of dust particles. A range of experimental studies have used electron microscopy to quantify the irregularity of dust particles. Most of these studies focused on measuring the aspect ratio of the particle, that is, the ratio of the particle’s major axis to its minor axis; the latter of these is usually obtained by fitting an ellipse to the particle and deriving the perpendicular dimension by requiring that the ellipsoid area equals that of the projected particle. These measurements indicate that the probability distribution function of the deviation of the aspect ratio from 1 (i.e., a perfect sphere) is well-described as a lognormal function. Measurements show that the median aspect ratio is in the range of 1.5 to 1.9, and the geometric standard deviation is approximately 0.6; both parameters are insensitive to particle size, although there’s an increasing trend in aspect ratio with transport distance due to the preferential settling of spherical dust. There are far fewer measurements of the particle’s third dimension, its height; the only extensive quantitative
measurements were reported by Okada et al.\textsuperscript{78}. They performed electron microscopy on dust sampled in China, and found that the ratio of particle height to minor axis is lognormally distributed, with a median of about 1/3. They also found only small variations in the average height-to-width ratio with particle size (see their Fig. 1b), and no clear relationship between aspect ratio and height-to-width ratio. Chou et al.\textsuperscript{79} also reported a height to major axis ratio of about one third, and Veghte and Freedman\textsuperscript{84} reported values ranging between 0.1 to 0.8 for different minerals.

Based on the measurements reviewed above, we describe the dust particle shape as a tri-axial ellipsoid\textsuperscript{60}, with the deviation of the aspect ratio (AR) from 1 (spherical) described by a lognormal function\textsuperscript{72} with a median aspect ratio of $\bar{\varepsilon}_a = 1.7 \pm 0.2$ and a geometric standard deviation of $\sigma_{\varepsilon_a} = 0.6 \pm 0.2$. That is,

$$P(AR) = \frac{1}{\sqrt{2\pi \sigma_{\varepsilon_a}}} (AR-1) \exp\left[ -0.5 \left( \frac{\ln(AR-1) - \ln(\bar{\varepsilon}_a-1)}{\sigma_{\varepsilon_a}} \right)^2 \right]$$ \hspace{1cm} (S.24)

The treatment of the dust particle height is impeded by two factors. First, there is only one detailed quantitative study of the probability distribution of dust particle heights\textsuperscript{78}, which also was taken of Asian dust rather than the globally more important North African dust. Second, the presence of very aspherical dust aerosols suggested by this study is difficult to account for, because the optical properties of highly aspherical particles are difficult to calculate and thus very uncertain\textsuperscript{85-88}, and are consequently not included in the Meng et al.\textsuperscript{60} single-scattering database. We therefore cannot realistically account for the distribution of particle heights, or its uncertainty, and instead take the height-to-length ratio as $\bar{\varepsilon}_h = 0.333$, based on the limited available measurements\textsuperscript{78, 79, 84}.

Note that we assume dust particles are randomly oriented\textsuperscript{85}, and that we cannot account for microscale roughness and thus have to assume dust particles are smooth.

2.3 Obtaining the globally-averaged extinction efficiency and its uncertainty

We use the single-scattering database of Meng et al.\textsuperscript{60} to convert the globally-averaged dust index of refraction and the ensemble of dust particle shapes to an ensemble of extinction efficiencies. We then take $Q_{\text{ext}}(D)$ as the average extinction efficiency of dust particles in this ensemble with a given geometric diameter $D$. For reference, we also calculated the extinction efficiency of spherical dust particles from Mie theory\textsuperscript{89} and included both curves in Figure 1b. As expected, the non-spherical shape of dust aerosols substantially increases their extinction efficiency over the case of spherical particles\textsuperscript{85, 90}.

To obtain the uncertainty in $Q_{\text{ext}}(D)$, we assumed that each of the parameters describing the refractive index of dust and the distribution of shapes is independent. This allowed us to obtain a large number of parameter sets by randomly choosing values from the normal distribution defined by the mean and standard error of each parameter as given above ($n = 1.53 \pm 0.03$; log($-k$) = -2.5 $\pm$ 0.3; $\bar{\varepsilon}_a = 1.7 \pm 0.3$, and $\sigma_{\varepsilon_a} = 0.6 \pm 0.2$). We then used the single-scattering database of Meng et al.\textsuperscript{60} to convert each set of parameters to a curve of $Q_{\text{ext}}(D)$, yielding a large number (i.e., $10^{5}$) of realizations of $Q_{\text{ext}}(D)$. We obtained the 95% confidence interval as the range within which 95% of these functions fall, which is given as the gray shading in Figure 1b. This confidence interval thus captures the uncertainty in the globally-averaged extinction efficiency due to the experimental uncertainty in the dust optical properties and in the probability distribution for the dust particle shape, as a function of dust geometric diameter.

3. Constraining the size-resolved global dust emission rate, loading, and DAOD

The previous sections described the procedure for constraining the globally-averaged dust PSD and extinction efficiency. Equations (1)-(4) combine these with constraints on the global dust aerosol optical depth (DAOD) from our companion study\textsuperscript{59}, yielding the size-resolved global atmospheric dust emission rate, and atmospheric mass loading $\frac{dN_{\text{atm}}}{dD}$ (Fig. 2b). Furthermore, the integrals over these quantities yield
the global dust emission rate $F_{\text{emit}}$ (Fig. 3a) and mass loading $L_{\text{atm}}$ (Fig. 3b). We also calculate the size-resolved DAOD from Eq. (2), yielding

$$\frac{d\tau_d}{dD} = \frac{L_{\text{atm}}}{A_{\text{Earth}}} \frac{dV_{\text{atm}}}{dD} \frac{A(D)}{M(D)} Q_{\text{ext}}(D).$$  \hspace{1cm} (S.25)

However, in order to use the analytical framework of equations (1)-(4) to constrain these quantities, we need to propagate the uncertainties in the various physical parameters and analytical functions. Due to the complexity of these analytical functions, the covariance of their parameters, and their occurrence inside of integrals, we cannot parametrically estimate the uncertainty on the desired quantities. We thus instead obtain their probability distribution functions (pdfs) using a non-parametric method based on the bootstrap method\textsuperscript{30, 31}, which is similar to the procedures described in Sections 1.1.2.3, 1.2.3, 1.3, and 2.3.

Indeed, the procedures above have yielded a large number of parameter sets for each of the functions used in Eqs. (1)–(4), namely for the globally-averaged size-resolved dust PSD ($\frac{dV_{\text{atm}}}{dD}$; see Section 1.3), the extinction efficiency ($Q_{\text{ext}}(D)$; see Section 2.3), and the dust lifetime ($T(D)$; see Section 1.2.3). Since these parameter sets are independent and identically distributed, we can apply the bootstrap technique to obtain pdfs of $\frac{dM_{\text{atm}}}{dD}$, $\frac{d\tau_d}{dD}$, $L_{\text{atm}}$, and $F_{\text{emit}}$. Specifically, we obtained these pdfs as follows:

1. We randomly selected a set of parameters from the bootstrap procedure performed on the functions $\frac{dV_{\text{atm}}}{dD}$, $Q_{\text{ext}}(D)$, and $T(D)$.
2. We randomly assigned a value to the dust density from the normal distribution defined by its mean and standard error $[\rho_d = (2.5 \pm 0.2) \times 10^3 \text{ kg/m}^3] $.
3. We used equations (2) – (4) to calculate the value of $c_e$.
4. We randomly chose a value of $\tau_d$ from its probability distribution obtained in Ridley et. al.\textsuperscript{59}.
5. We used equation (1) to obtain $L_{\text{atm}}$.
6. We used that $T = L_{\text{atm}} / F_{\text{emit}}$ to obtain $F_{\text{emit}}$.
7. We used the value of $L_{\text{atm}}$ to obtain $\frac{dM_{\text{atm}}}{dD} = L_{\text{atm}} \frac{dV_{\text{atm}}}{dD}$.
8. We used Eq. (S.25) to obtain $\frac{d\tau_d}{dD}$.
9. We repeated the procedure in steps 1-8 a large number of times (i.e., $10^5$).

Since the resulting distribution of values of $\frac{dM_{\text{atm}}}{dD}$, $\frac{d\tau_d}{dD}$, $L_{\text{atm}}$, and $F_{\text{emit}}$ correspond to their probability distributions\textsuperscript{30, 31}, we obtained the 95% confidence intervals from the range in which 95% of the obtained values lie (grey shading in Fig. 2 and blue shading in Fig. 3).

4. Constraining the dust direct radiative effect (DRE)

The dust direct radiative effect is generated by extinction of radiation. It therefore is closely connected to the globally-averaged dust aerosol optical depth $\tau_d$ (e.g., Ref. \textsuperscript{91}), which is a measure of the global extinction of SW radiation by dust. Consequently, we constrain the dust DRE by combining results on the size-resolved DAOD (Fig. 2c) with simulations of the efficiency with which this extinction produces DRE at top-of-atmosphere (TOA). Specifically, we have that

$$\zeta = \zeta_{\text{SW}} + \zeta_{\text{LW}} = \int_0^D \frac{d\tau_d}{dD} \Omega_{\text{SW}}(D)dD + \int_0^D \frac{d\tau_d}{dD} \Omega_{\text{LW}}(D)dD, \hspace{1cm} (S.26)$$

where $\zeta_{\text{SW}}$ and $\zeta_{\text{LW}}$ denote the SW and LW contributions to the total DRE ($\zeta$), and the radiative effect efficiencies $\Omega_{\text{SW}}(D) = \frac{d\zeta_{\text{SW}}}{d\tau_d}$ and $\Omega_{\text{LW}}(D) = \frac{d\zeta_{\text{LW}}}{d\tau_d}$ are the all sky DRE that dust of diameter $D$ produces per unit DAOD, due to interactions with SW and LW radiation, respectively.

The values of $\Omega_{\text{SW}}(D)$ and $\Omega_{\text{LW}}(D)$ depend on numerous factors, including the spatial and temporal variability of dust, the surface albedo and surface emissivity, the vertical temperature profile, the distribution of radiatively-active species such as clouds and greenhouse gases, and the asymmetry parameter and single-scattering albedo of dust. We thus require global model simulations to estimate
\[ \Omega_{SW}(D) \] and \[ \Omega_{LW}(D) \], for which we use results from four leading global models, namely CESM, GISS, GEOS-Chem, and WRF-Chem (see Fig. S3). The CESM simulations are described in Kok et al.\(^2\), with dust optical properties from Albani et al.\(^3\), and the methodology for obtaining the radiative effects for each particle bin are described in Conley et al.\(^9\). The GISS simulations are described in Miller et al.\(^2\) (see especially their Fig. 2), and the WRF-Chem and GEOS-Chem simulations are described in Sections 5.1 and 5.2, respectively. All simulations use dust absorption properties that are consistent with recent findings that dust is less absorptive in the SW spectrum than previously thought\(^74, 94, 95\).

Most global models unfortunately do not account for the radiative effects of scattering of LW radiation. At TOA, the DRE from LW scattering likely accounts for about half\(^6\) of the total LW DRE for a variety of standard clear-sky conditions. However, we follow the conservative treatment of Miller et al.\(^2\) in assuming that LW scattering enhances the radiative effect from LW absorption by a factor of \(\beta_{LW,scat} = 0.3\), thus accounting for only 23\% of the LW DRE at TOA. As such, the constraints on LW warming by dust obtained here should be seen as conservative.

We aim to use Eqs. (S.27, S.28) to compute the DRE that would be produced by the atmospheric dust size distribution simulated by the seven AeroCom models (colored lines in Fig. 2b). To do so, we require the optical depth \(\Delta \tau_{d,i,k}\) that the mass size distribution of AeroCom model \(j\) would yield for particle bin \(k\) of global model \(i\). We calculated \(\Delta \tau_{d,i,k}\) as follows\(^36, 97\):

\[
\Delta \tau_{d,i,k} = \frac{1}{A_{Earth}} \int_{D_{i,j}}^{D_{i,j}} M_j(D) \frac{dM_j}{dD} Q_{ext,sph}(D) dD,
\]

where the surface area-to-mass ratio \(A(D)/M(D) = 3/2\rho_a D\) (see Eq. 2). We obtained the mass size distribution for AeroCom model \(j\), \(\frac{dM_j}{dD}\), by fitting power laws between the individual values of \(\frac{dM_j}{dD}\) given by each particle bin; these power laws correspond to the solid colored lines in Fig. 2b. The extinction coefficient \(Q_{ext,sph}\) was taken as that of spherical dust, consistent with the assumption of spherical dust made in AeroCom models\(^2, 44-46\), and thus calculated with Mie theory (brown line in Fig. 1b). To ensure that the results are consistent with reported AeroCom results, we normalized the sum of the optical depths.
of the particle bins with the total optical depth $\tau_{d,j}$ for the AeroCom model $j$, given in Table 3 of Huneeus et al.\textsuperscript{98}. That is,

$$\Delta \tau_{d,i,k}^j = \Delta \tau_{d,i,k}^j \frac{\sum_k \Delta \tau_{d,i,k}^j}{\tau_{d,j}}$$  \hspace{1cm} (S.30)

where $\Delta \tau_{d,i,k}^j$ is the normalized optical depth for each particle bin. We inserted $\Delta \tau_{d,i,k}^j$ into Eq. (S.27) to calculate the DRE for each particle bin of each combination of global model $i$ and AeroCom model $j$.

These results are shown in Figs. S4a-d for the four global models CESM, GISS, GEOS-Chem, and WRF-Chem; the error bars in these figures represent the spread from using $\Delta \tau_{d,i,k}^j$ from the seven AeroCom models. We also use a similar procedure to Eqs. (S.29) and (S.30) to calculate the size-resolved dust optical depth (i.e., $d\tau_d/dD$) for each AeroCom model, which is plotted in Fig. 2c.

We combined the above procedure with Eq. (S.28) to obtain a total of 28 estimates of the total TOA dust DRE, which resulted from the combination of seven AeroCom models with four global model calculations of $\Omega(D)$. Fig. 4 shows the median of these 28 values and the 95% CI, which we estimated from the range of the 26 values remaining after eliminating the two extreme values. Fig. 4 also shows six published estimates of DRE from AeroCom models\textsuperscript{2, 74, 95, 99}, although two of the three estimates in Forster et al.\textsuperscript{95} include the SW radiative effect only.

4.2 Calculating the DRE using this study’s constraints on the size-resolved DAOD

In addition to using the size-resolved dust loading from AeroCom simulations to calculate the dust DRE, we use the constraints on $\Delta \tau_{d,i,k}$ from our analysis (see Figs. 2c and S1c) to calculate the DRE. Specifically, we inserted a large number ($10^5$) of realizations of $\Delta \tau_{d,i,k}$ (see Section 3) into Eq. (S.27) to obtain the pdfs of $\chi_{i,k,SW}$ and $\chi_{i,k,LW}$, for which the median and the 95% CI are plotted in Figs. S4e-h. The difference of the radiative effect per particle bin with those calculated using the size-resolved dust loading from AeroCom models is shown in Figs. S4i-l. We then inserted these $10^5$ realizations of $\chi_{i,k,SW}$ and $\chi_{i,k,LW}$ into Eq. (S.28) to obtain the pdfs of $\zeta_{SW}$ and $\zeta_{LW}$ (see Figs. S5a, b). Finally, we used $\zeta_{SW}$ and $\zeta_{LW}$ in the following procedure to obtain the pdf of the total DRE ($\zeta$):

1. Obtain a realization of $d\tau_d/dD$ (see Section 3).
2. Randomly pick one of the four global models providing $\Omega_{SW}(D)$ (Fig. S3a) and randomly select a corresponding realization of $\zeta_{SW}$.
3. Randomly pick one of the four global models providing $\Omega_{LW}(D)$ (Fig. S3b) and randomly select a corresponding realization of $\zeta_{LW}$.
4. Obtain a realization of the total DRE from $\zeta = \zeta_{SW} + \zeta_{LW}$.
5. Repeat steps 1-4 a large number of times ($10^5$) to obtain the pdf of $\zeta$, which is plotted in Fig. S5c.

We report the median and 95% CI of the probability distributions of $\zeta_{SW}$, $\zeta_{LW}$, and $\zeta$ in Fig. 4.

5. Global transport model simulations

To supplement the size-resolved dust lifetimes that have been reported in the literature\textsuperscript{32-36, 40}, we performed simulations with a number of leading global transport and climate models, namely WRF-Chem, GEOS-Chem, and HadGEM. Simulations with the first two models were also used to supplement the size-resolved radiative effect efficiencies obtained from previously-reported simulations with CESM\textsuperscript{34} and GISS\textsuperscript{3}. We describe the simulations with these three models below.

5.1 Description of WRF-Chem simulations

We used WRF-Chem version 3.5.1, updated by the Pacific Northwest National Laboratory (PNNL), to simulate the dust aerosol lifetime (Fig. 1d) and the size-resolved radiative effect efficiency (Fig. S3), averaged over the years 2004-2008. Our simulations used the MOSAIC (Model for Simulation Aerosol Interactions and Chemistry) aerosol module\textsuperscript{100} coupled with the CBM-Z (carbon bond mechanism) photochemical mechanism\textsuperscript{101}. The MOSAIC aerosol scheme uses the sectional approach to represent aerosol size distributions with eight discrete size bins\textsuperscript{102}, extending to 10 µm diameter. All major aerosol components, including sulfate (SO$_4^{2-}$), nitrate (NO$_3^-$), ammonium (NH$_4^+$), black carbon (BC), organic
matter (OM), sea-salt, methanesulfonic acid, and mineral dust are simulated in the model. The MOSAIC aerosol scheme includes physical and chemical processes of nucleation, condensation, coagulation, aqueous phase chemistry, and water uptake by aerosols. The treatment of dry and wet deposition processes are described in Refs. 103, 104.

In WRF-Chem, aerosol optical properties such as extinction, single-scattering albedo (SSA), and asymmetry factor for scattering are computed as a function of wavelength for each model grid box. Aerosols are assumed internally mixed in each bin, i.e., a complex refractive index is calculated by volume averaging for each bin for each chemical constituent of aerosols. The Optical Properties of Aerosols and Clouds (OPAC) dataset105 is used for the SW and LW refractive indices of aerosols, except that a constant value of 1.53±0.003i is used for the SW refractive index of dust following Zhao et al.106, 107, which is consistent with recent insights62, 63, 74, 94. A detailed description of the computation of aerosol optical properties in WRF-Chem can be found in Fast et al.102 and Barnard et al.108. Aerosol radiative feedback is coupled with the Rapid Radiative Transfer Model (RRTMG)109, 110 for both shortwave (SW) and longwave (LW) radiation107. Since aerosols in WRF-Chem are assumed internally mixed, the optical properties and direct radiative forcing of individual aerosol species in the atmosphere is not explicitly calculated. Instead, the methodology described in Zhao et al.104 is used to diagnose the optical depth and direct radiative effect of individual aerosol species. Therefore, large uncertainties in estimating radiative effects of one individual aerosol species can be introduced in the case of very low mass concentrations. In this study, we found that dust mass concentrations in the first three bins (0.039 – 0.078, 0.078 – 0.156, and 0.156 – 0.312 µm) are quite low. Since such low concentrations produce large relative uncertainties in estimating the particle bin’s radiative effects, we omitted those bins in our calculation of DRE using the WRF-Chem simulations.

Following Zhao et al.103, we used a quasi-global channel configuration (using periodic boundary conditions in the zonal direction) with 360 × 145 grid cells (180° W-180° E, 67.5° S-77.5° N) to perform the simulation at 1° horizontal resolution over the period of 2004-2008. The simulations are configured with 35 vertical layers up to 50 hPa. The meteorological initial and lateral boundary (only for the meridional direction) conditions are derived from the National Center for Environmental Prediction final analysis (NCEP/FNL) data at 1° horizontal resolution and 6 h temporal intervals. The modeled wind components and atmospheric temperature are nudged towards the NCEP/FNL reanalysis data with a nudging timescale of 6 hr111. The chemical initial and boundary (only for the meridional direction) conditions are taken from the default profiles in WRF-Chem, which are the same as those used by McKeen et al.112 and are based on averages of mid-latitude aircraft profiles from several field studies over the eastern Pacific Ocean. This study uses a set of selected schemes for model physics, including the MYJ (Mellor–Yamada–Janjic) planetary boundary layer scheme, Noah land surface scheme, Morrison 2-moment microphysics scheme, Kain-Fritsch cumulus scheme, and RRTMG longwave and shortwave radiation schemes.

Vertical dust emission fluxes are calculated with the Goddard Chemical Aerosol Radiation Transport (GOCART) dust emission scheme45, and the emitted dust particles are distributed into the MOSAIC aerosol size bins following the theoretical expression of Kok10.

5.2 Description of GEOS-Chem simulations

We used simulations with GEOS-Chem (version v9-01-03; http://www.geos-chem.org/) for the years 2004 – 2008 to simulate both the dust aerosol lifetime (Fig. 1d) and the size-resolved radiative effect efficiency (Fig. S3). The GEOS-Chem model incorporates a global three-dimensional simulation of coupled oxidant–aerosol chemistry, run at a resolution of 2° ×2.5° latitude and longitude, and 47 vertical levels. The model is driven by assimilated MERRA meteorology from the Goddard Earth Observing System of the NASA Global Modeling and Assimilation Office (GMAO), including assimilated meteorological fields at 1-hourly and 3-hourly temporal resolution. The aerosol types simulated include sulfate–nitrate–ammonium aerosols113, and carbonaceous aerosols114-116, mineral dust16, 117 and sea salt118. Dust emission in GEOS-Chem is based upon the DEAD dust scheme46, making use of the GOCART source function45. Mineral dust mass is transported in four different sized bins (0.1–1.0, 1.0–1.8, 1.8–3.0
and 3.0–6.0 μm), the smallest of which is partitioned into four bins (0.10–0.18, 0.18–0.30, 0.30–0.65 and 0.65–1.00 μm) when deriving optical properties, owing to the strong size dependence of extinction for sub-micron aerosol. Dust emission is modified from the standard model to treat 10-m wind fields as a Weibull distribution based on sub-grid wind statistics. Aerosol optical depth (AOD) is calculated online assuming log-normal size distributions of externally mixed aerosols and is a function of the local relative humidity to account for hygroscopic growth. Aerosol optical properties employed here are based on the Global Aerosol Data Set (GADS) with modifications to the size distribution based on field observations, and modifications to the dust refractive indices to match the observed lower SW absorption. The refractive indices for each species are interpolated to the 30 wavelengths, between 230 nm and 56 μm, used by the radiative transfer code (RRTMG) coupled with GEOS-Chem. Surface albedo and emissivity are generated from MODIS (MOD11C2 and MCD43C3 products) to provide 8-day averages for a climatology between 2002 and 2007. Cloud optical properties are calculated based on liquid and ice optical depths from the MERRA meteorology with cloud overlap treated using the Monte Carlo independent column approximation (McICA). The radiative transfer code is run twice every 3 hours in the simulation, once to calculate the flux with all aerosol and a second time with only non-dust aerosol. The difference yields the direct radiative effect (DRE) of dust aerosols.

5.3 Description of HadGEM simulations

The design and the implementation of the UK MetOffice HadGEM2 model family is described in Martin et al. in detail. For our experiment, we simulated the dust aerosol lifetime (Fig. 1d) using the HadGEM2-A model version, which is the atmosphere only version of the global model, run with prescribed SSTs and sea ice climatology updated every 24 hours. Both are based on the Reynolds SST Analysis, averaged over the 1995-2005 period. The vertically-extended HI-TOP version of the model was used, with 85 levels extending to 85km height. The horizontal resolution is 1.25 degrees (latitudinal) by 1.875 degrees (longitudinal), which produces a global grid of 192 x 145 grid cells (N96). This is equivalent to a surface resolution of about 208x139 km² at the Equator, reducing to 120x139 km² at 55 degrees of latitude.

Six aerosol species are incorporated in the model using the CLASSIC aerosol scheme: sulfate, black carbon, biomass burning elemental carbon, fossil fuel organic carbon, mineral dust, and sea salt aerosols. We use monthly averages of Atmospheric Optical Depth (AOD) at 550 nm wavelength for each component, which is available as a prognostic model quantity, except for Sea Salt when it is a diagnosed quantity. Emission datasets for aerosol precursors and primary aerosols have been revised with the HadGEM2 family, using datasets created in support of CMIP5.

The models dust emission scheme has remained unchanged compared to earlier versions of HadGEM. It is based on the widely used emission parameterization developed by Marticorena and Bergametti. The horizontal flux is calculated for 9 model size bins with boundaries at 0.0316, 0.1, 0.316, 1, 3.16, 10, 31.6, 100, 316 and 1000 μm radius. The vertical emission flux is calculated for 6 model size bins in the size range between 0.0316 to 31.6 μm radius (same bin intervals). The horizontal-to-vertical-mass flux ratio is assumed to be a constant of proportionality as a function of particle size. The mass fraction of particles in each size bin is calculated off-line from the clay, silt and sand fraction data from the International Geosphere-Biosphere Programme (IGBP) global soil data. The threshold friction velocity is also fixed for each size bin. Soil moisture and roughness corrections follow the method of Fecan et al. and Marticorena and Bergametti, respectively.

In order to constrain the dust emission flux over major source regions, the concept of preferential sources that vary as a function of topography is applied. Once emitted, the dust aerosols are treated as independent tracers in the atmosphere, such as all the other aerosol species. Sedimentation and turbulent mixing are considered as dry removal mechanisms of dust particles from the atmosphere. Wet removal due to precipitation scavenging within and below cloud for both large-scale and convective precipitation is included using a first-order removal rate. Finally, dust-radiation interaction through SW and LW direct effects is permitted in the model.
References
30. Efron, B. *The jackknife, the bootstrap, and other resampling plans*. (Capital City Press, Montpelier, Vermont; 1982).


89. Mätzler, C. (Institut für Angewandte Physik, Bern, Switzerland; 2002).


134. Woodward, S. in Hadley Centre Technical Note 87 (Met Office Hadley Centre; 2011).