Supplementary Figure 1 | CNT mechanical transfer (a) Schematics showing steps of pressing down and retracting during the transfer of the CNT from the growth support to the electrodes. (b) Optical image of the forks aligned and pressed over source drain contacts during the transfer process. Pillar structure in the recess with source (S), drain (D) and bottom gates (LG, MG, RG) can also be seen. (c) SEM image of the silicon forks with CNT grown on it. (d) SEM image of a transferred CNT on the source drain contacts. CNTs are marked by the solid arrows.

Supplementary Figure 2 | Measurement setup Simultaneous measurements of DC and reflectometry are enabled by a bias-tee attached at the input port of the stub tuner. The directional coupler isolates the attenuated (by ~ 55 dB) input signal from the reflected one by 20 dB. The latter is amplified by first a cryogenic amplifier and then a room temperature amplifier before being measured by a vector network analyzer (VNA).
Supplementary Figure 3 | Resonance response to conductance changes (a) Measured stub tuner resonance response for different CNT conductances ($I/V$) measured at a fixed bias of 20 mV and different gate voltages show a reduction in the power response on increasing conductance. Negligible changes in the linewidth reflect that circuit response is limited by the internal losses of the Nb. Black curve is a fit using eq. 3 of the main text. (b) Simultaneously measured phase response with panel (a). Asymmetry around zero is due to uncalibrated linear phase changes from coax cable. (c) Calculated resonance depth (solid line) versus conductance for parameters extracted from the fit at zero conductance in the panel (a). The response in the lossless case (dashed curve) for the same device parameters as in the black solid curve show a matching near $G_{\text{CNT}} \approx 3.2 \mu\text{S}$. (d) Calculated resonance depth for the designed lengths with the same loss factor as in panel (a) shows an optimal power matching for $G_{\text{CNT}} \approx 5 \mu\text{S}$.

Supplementary Figure 4 | Impedance matching in a single quantum dot (a) Measured $\Gamma$ at a temperature of 3 K and a probe power of $-90 \text{ dBm}$ in the device B for different $dI/dV$, showing matching down to $-40 \text{ dB}$ at a conductance of $3.9 \mu\text{S}$ and bandwidth (BW) up to 2 MHz. Black curve is a fit using eq. 3 of the main text. (b) Calculated resonance depth versus conductance for parameters extracted from the fit at zero conductance in the panel (a). (c) Differential conductance measurement of the single quantum dot, obtained by numerical differentiation. The two lines at $V_{\text{SD}} = \pm 11 \text{ mV}$ correspond to a range switching of the voltage source. (d) Extracted $G$ (real part of the admittance) from the resonance curves taken at each $V_{\text{SD}}/V_G$ point, showing a similar result as in (b).
Supplementary Figure 5 | Tuning inter-dot coupling between the double quantum dots DC measurement of a p-p double dot regime for different values of $V_{MG}$ at $V_{SD} = -5$ mV. Increasing the middle gate voltage $V_{MG}$ to positive values increases the barrier between the dots thereby decreasing the tunnel coupling. Note that corresponding voltages for charge triple points are different for different graphs due to confinement potential being affected by all gates.

Supplementary Figure 6 | Reflectometry response at interdot coupling between (1,2) and (2,1) hole states. (a) Extracted $t_c$ and $\Gamma_{tot}$ from the phase response show a similar behavior to the one shown in the main text in Fig. 4f. (b) Simultaneous measurement of the reflected power, phase and current at $V_{MG} = 270$ mV show no frequency shift due to large $\Gamma_{tot}$. The change in the amplitude is due to near-resonant absorption from the hybridized double dot rather than conductance changes (see also the main text). Error bars are smaller than the symbol sizes in panel (a).

Supplementary Note 1 In the following, we provide additional information about the measurements discussed in the main text. Measurements on another device (B) are also presented here. Unless explicitly stated, the details of measurements or calculations correspond to the device described in the main text.

Supplementary Note 2 | Mechanical transfer of CNT. Carbon nanotubes are grown on polycrystalline silicon fork-like structures using iron-loaded ferritin proteins as catalysts [1]. The poly-Si arms are 2 µm wide and the gap spanned by the CNTs ∼ 8 µm. Growth is done in a chemical vapor deposition chamber maintained at CH$_4$ + H$_2$ (155 mbar + 65 mbar) atmosphere and 850 °C for 15 minutes. Reference forks with CNTs are imaged under SEM to assess the density of the tubes and subsequently optimize the catalyst concentration. A low density is needed to ideally have one nanotube per fork.

The forks with pristine CNTs, not exposed to any electron beam or post-processing, are mounted on a clamp and aligned with the source-drain contacts using the optical microscope of a micro-manipulator setup. This is followed by pressing the CNT onto the contacts using three-axis piezo control. Since the device is already bonded, successful transfer can be monitored through voltage biased (200 mV) resistance measurements. Voltages are then applied to the gates to determine the metallic or semiconducting nature.
of the tube. The forks are carefully retracted leaving the nanotube held on the source drain contact due to van der Waals force. We aim at good transparent contacts, meaning a low resistance. However, during the transfer the resistance is sometimes too low indicating that two or more CNTs have been transferred. One can simply apply a large bias voltage to remove all CNTs at once and try again. The assembly hence allows the reuse of the same circuit with different tubes. The steps of the transfer are schematically presented in the Supplementary Fig. 1.

**Supplementary Note 3 | Measurement setup.** The device is measured in a dilution refrigerator with a base temperature of 20 mK. Due to the high device impedance $\sim 100 \, \text{M} \Omega$ and a $2 \, \text{nF}$ shunt capacitance of the DC line pi-filters, differential conductance measurements could not be performed. Instead DC currents are measured. RF measurements of amplitude and phase are performed with a vector network analyzer (VNA). Details of the setup are shown in Supplementary Fig. 2.

**Supplementary Note 4 | Stub tuner response.** In contrast to circuit quantum electrodynamics (cQED) experiments where half-wave resonators are typically utilized, we implement a transmission line based impedance matching circuit, termed stub tuner [2]. It consists of two pieces of transmission lines connected in parallel, with the device placed on one end, the other ending in an open circuit (Fig. 1a). This type of circuit is typically utilized to transform the impedance of an arbitrary load to the characteristic impedance of a feedline, but can also be used to perform reflectometry on a variable load. In a simple analogy, the device is an interferometer, where the line lengths determine the resonance frequency, the difference in length determines $Z_{\text{Match}}$ and $\alpha$ determines the sensitivity of the circuit to changes in the load. The device can be completely described by five parameters: $l, d, \epsilon, \alpha$ and $Z_0$ (see main text). Except $\alpha$, all other parameters are geometry and substrate dependent which can be be chosen in the design stage. Furthermore, stray capacitances and inductances in the microwave device only change the effective length of the transmission lines [3], which can be compensated for in the design.

In the device described in the main text, the matched load is affected by intrinsic loss ($\alpha = 0.0074 \, \text{m}^{-1}$) of the stub tuner ($G_{\text{Match}} \approx 1.6 \, \mu \text{S} < G_{\text{Loss}} \approx 3.2 \, \mu \text{S}$). This leads to a loss limited resonance and a monotonic decrease of the reflection coefficient as function of load (Supplementary Fig. 3a, 3c). We note that a partial reason for the circuit to be in the loss dominated regime is due to the parasitic inductances that change the effective length of the transmission lines. Fitting the resonance for zero conductance using eq. 1 in the main text, we determined the lengths $l = 10.56 \, \text{mm}$ and $d = 10.44 \, \text{mm}$ to be different from planned geometrical lengths $10.66 \, \text{mm}$ and $10.36 \, \text{mm}$, respectively. If the final effective lengths had been the same as the geometric ones in the device, one would have an effective $G_{\text{Match}} \approx 8 \, \mu \text{S}$ for zero losses. The calculated reflectance response at resonance for $\alpha = 0.0074 \, \text{m}^{-1}$ ($G_{\text{Loss}} \approx 3.2 \, \mu \text{S}$) is shown in Supplementary Fig. 3d, clearly showing the circuit to be in the load dominated regime with optimal power matching at $G_{\text{CNT}} \approx 5 \, \mu \text{S}$.

**Supplementary Note 5 | Quantitative parameter extraction.** Two features that separate the stub tuner from a half-wave resonator are the comparative simple extraction of the RF device impedance and the enhanced coupling of microwaves into and out of the device in a specific frequency interval. The circuit offers bandwidths in the MHz range even for device impedances on the order of $1 \, \text{M} \Omega$, which allows for a quantitative determination of the device impedance with potentially sub-$\mu \text{s}$ time-resolution, limited by the obtainable signal to noise ratio.

We demonstrate full impedance matching in a second device B fabricated using the same mechanical transfer. The device is similar to the one discussed in the main text, with a few differences. There is only one bottom gate and it is vertically separated from the source-drain contacts by $2 \, \mu \text{m}$. The latter significantly reduces the gate lever arm. The stub tuner is patterned on a NbTiN film (on a sapphire substrate) with lengths $l = 5.498 \, \text{mm}$ and $d = 5.33 \, \text{mm}$ operating near $5 \, \text{GHz}$. Due to larger kinetic inductance, the transmission line has a higher $Z_0 = 70 \, \Omega$ compared to Nb ($50 \, \Omega$) for a central conductor width of $10 \, \mu \text{m}$ and gap width of $5.5 \, \mu \text{m}$. From the reflection spectrum taken in the bandgap, we extract $\alpha = 0.0016 \, \text{m}^{-1}$ and lengths within $30 \, \mu \text{m}$ of the designed values. Low loss ($G_{\text{Match}} > G_{\text{Loss}}$) and reduced parasitic inductances in this case put the circuit in the load dominated regime with a matched load of $\approx 4 \, \mu \text{S}$ (Supplementary Fig. 4b). Parasitic inductances arise from connections of source (drain) contacts to the central conductor (ground plane) of stub-tuners which deviate from a $50 \, \Omega$ transmission line geometry. The layout of device B aims at minimizing such deviations by keeping the connections short $\approx 10 \, \mu \text{m}$ compared to the one discussed
in the main text \(\approx 100 \mu m\).

At negative gate voltages and high bias (\(\approx 200 \text{ mV}\)), the differential conductance of the device is large enough to be close to the matched load. A series of spectra taken at different bias voltages is shown in Supplementary Fig. 4a, showing an increase of the resonance depth as \(dI/dV\) increases, with a maximum depth of 40 dB at 3.9 \(\mu S\). We furthermore achieve comparatively large bandwidth of \(\approx 2 \text{ MHz}\) close to the match. This is in agreement with the numerically approximated value of \((4/\pi) f_r Z_0 G_{\text{Match}}\) derived from the eq. 1 of the main text for lossless case. We attribute the frequency shift of \(\approx 140 \text{ kHz}\) at large \(dI/dV\) to the quantum capacitance, which we determine to be \(\approx 35 \text{ aF}\) [4].

We now map a number of Coulomb diamonds by sweeping the bias \(V_{\text{SD}}\) and gate \(V_G\) voltages, and at each point acquire a reflection spectrum as well as the direct current. Fixing the extracted stub-tuner parameters acquired from the bandgap resonance response, we extract the differential conductance \(G\) from each spectrum of the gate-bias map. Here we assume that \(G\) does not change over few MHz around the resonance frequency and extracted quantity represents \(G\) at 5 GHz. The numerical \(dI/dV\) obtained from the DC measurement and extracted \(G\) are plotted in in Supplementary Fig. 4c,d. We obtain quantitatively very similar plots, namely Coulomb diamonds with a charging energy of approximately 10 meV, comparable to the expected value for a suspended tube of 650 nm length. Minor deviations in two plots near the left-bottom diamond edges are due to relatively smaller sensitivity of the resonance fits at conductances far from matching.

**Supplementary Note 6 | Measurements and tunability of inter-dot coupling.** In this section we discuss the details of the measurements involving the tunability of the tunnel coupling between the double dots. We go to the p-p double dot regime and increase the middle gate \(V_{\text{MG}}\) to positive values to raise the barrier between hole dots. In our device, the confinement potential is affected by all the local gates. Consequently, a change in \(V_{\text{MG}}\) also shifts the charge triple points corresponding to a specific dot state. For example, in the measurements shown in Fig. 4 (main text), both \(V_{\text{LG}}\) and \(V_{\text{RG}}\) shift to more negative voltages. Quantitatively, a change in \(V_{\text{MG}}\) by 100 mV causes both \(V_{\text{LG}}\) and \(V_{\text{RG}}\) to shift by approximately 60 mV each (see Supplementary Fig. 5). We take density plots (shown in Fig. 4c in the main text) at every \(V_{\text{MC}}\) to track the shift and take a reflectometry scan in the middle of the two triple points. We then fit the scan to eq. 11 to obtain tunnel coupling and dephasing rates of the double-dot qubit. Supplementary Fig. 6a shows extracted \(t_c\) and \(\Gamma_{\text{tot}}\) for charge degeneracy between (1,2) and (2,1) hole states. Their behavior versus \(V_{\text{MG}}\) is found very similar to those between (2,2) and (1,3) hole states described in Fig. 4 of the main text.

**Supplementary Note 7 | Dipole coupling of hybridized dots to a microwave resonance.** Though our device is DC coupled, we work in a regime with zero current for extracting the inter-dot coupling energy. Consequently, we can consider the system similar to a standard microwave resonator capacitively coupled to a qubit formed by two charge states on a (nanotube) double quantum dot. The Hamiltonian of this system is a variant of the well-known Jaynes-Cummings model

\[
H_0 = \omega_d \hat{a}^\dagger \hat{a} + \omega_\text{q} \hat{\sigma}^+ \hat{\sigma}^- + g (\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+ ) \tag{1}
\]

where the qubit energy is \(\omega_\text{q} = \sqrt{\delta^2 + 4D^2}\), the resonator-qubit coupling is \(g = g_0 \sin \vartheta\), and the mixing angle is \(\sin \vartheta = 2t_c / \omega_d\). In these expressions, \(\hbar\) is set to unity, \(\delta\) the detuning between the two relevant charge states and \(t_c\) the tunneling amplitude.

Including the microwave drive and in the rotating frame of the drive the Hamiltonian reads

\[
H = -\Delta_r \hat{\sigma}^\dagger \hat{a} - \Delta_\text{q} \hat{\sigma}^+ \hat{\sigma}^- + g (\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+ ) + \Omega (\hat{a} + \hat{a}^\dagger) \tag{2}
\]

where we introduced the driving strength \(\Omega\) and the detuning of the microwave drive from the resonator \(\Delta_r = \omega_{\text{drive}} - \omega_d\), and the qubit \(\Delta_\text{q} = \omega_{\text{drive}} - \omega_\text{q}\).

Additionally, we take into account resonator \((\kappa)\) and qubit relaxation \((\gamma)\) as well as qubit dephasing \((\Gamma_\varphi)\) rates. These can be described with a Markovian master equation approach with

\[
\dot{\varrho} = -i[H, \varrho] + \kappa D[\hat{a}] \varrho + \frac{\Gamma_\varphi}{2} D[\hat{\sigma}^\dagger \hat{\sigma}] \varrho + \gamma (n_{\text{th}} + 1) D[\hat{\sigma}^-] \varrho + \gamma n_{\text{th}} D[\hat{\sigma}^+] \varrho \tag{3}
\]

\[
+ \gamma (n_{\text{th}} + 1) D[\hat{\sigma}^-] \varrho + \gamma n_{\text{th}} D[\hat{\sigma}^+] \varrho \tag{4}
\]
where $\mathcal{D}[\hat{A}] = \hat{A}\hat{D} - \{\hat{A}^\dagger, \hat{D}\}/2$ is the Lindblad dissipator and average photon number at temperature $T$ given by $n_{\text{th}} = [e^{\hbar\omega_d/(k_B T)} - 1]^{-1}$. This gives rise to the following equations of motion

\begin{equation}
\langle \dot{\hat{a}} \rangle = +i\Delta_r \langle \hat{a} \rangle - \frac{\kappa}{2} \langle \hat{a} \rangle - i\Omega - ig\langle \hat{a} \hat{a}^\dagger \rangle \tag{5}
\end{equation}

\begin{equation}
\langle \dot{\hat{a}}^\dagger \rangle = +i\Delta_d \langle \hat{a}^\dagger \rangle - \left( \frac{\gamma}{2} + \Gamma_\phi \right) \langle \hat{a}^\dagger \rangle + ig\langle \hat{a} \hat{a}^\dagger \hat{\sigma}_z \rangle \tag{6}
\end{equation}

In a semiclassical decoupling approximation, i.e. assuming $\langle \hat{a} \hat{a}^\dagger \hat{\sigma}_z \rangle \approx \langle \hat{a} \rangle \langle \hat{\sigma}_z \rangle$, we can solve for the steady state

\begin{equation}
\langle \hat{\sigma}_z \rangle_{\text{ss}} = \frac{-ig\langle \hat{a} \rangle \langle \hat{\sigma}_z \rangle}{\frac{\gamma}{2} + \Gamma_\phi - i\Delta_d} \tag{7}
\end{equation}

and we then obtain

\begin{equation}
\langle \hat{a} \rangle_{\text{ss}} = \frac{-i\Omega}{\frac{\gamma}{2} - i\Delta_r - \frac{g^2\langle \hat{\sigma}_z \rangle}{(\frac{\gamma}{2} + \Gamma_\phi - i\Delta_d)} \tag{8}
\end{equation}

where we implicitly assumed that the coupling to the driven resonator does not change the qubit polarization $\langle \hat{\sigma}_z \rangle$, i.e. the qubit remains in thermal equilibrium with the electronic bath and $\langle \hat{\sigma}_z \rangle = \frac{1}{2} n_{\text{th}} - 1$. This gives rise to the following equations of motion

\begin{align}
\langle \dot{\hat{a}} \rangle &= +i\Delta_r \langle \hat{a} \rangle - \frac{\kappa}{2} \langle \hat{a} \rangle - i\Omega - ig\langle \hat{a} \hat{a}^\dagger \rangle \\
\langle \dot{\hat{a}}^\dagger \rangle &= +i\Delta_d \langle \hat{a}^\dagger \rangle - \left( \frac{\gamma}{2} + \Gamma_\phi \right) \langle \hat{a}^\dagger \rangle + ig\langle \hat{a} \hat{a}^\dagger \hat{\sigma}_z \rangle
\end{align}

If the system remains in the weak coupling limit, $g \ll \gamma$, this expression is well approximated by

\begin{equation}
\langle \hat{a} \rangle_{\text{ss}} = \frac{-i\Omega}{\frac{\gamma}{2} - i\Delta_r + i\chi \langle \hat{\sigma}_z \rangle} \tag{9}
\end{equation}

where we have introduced the susceptibility

\begin{equation}
\chi = +i \frac{g^2}{\left( \frac{\gamma}{2} + \Gamma_\phi \right) - i\Delta_d} \approx \frac{g^2}{-i \left( \frac{\gamma}{2} + \Gamma_\phi \right) + (\omega_d - \omega_r)}. \tag{10}
\end{equation}

Its real and imaginary parts lead to qubit-dependent shifts of the resonator frequency $\omega_r$ and the resonator linewidth $\kappa$

\begin{align}
\Delta\omega_R &= \text{Re}[\chi] \langle \hat{\sigma}_z \rangle = \frac{g^2 \Delta \langle \hat{\sigma}_z \rangle}{\Gamma_{\text{tot}}^2 + \Delta^2} \tag{11}
\\
\Delta\kappa &= -2\text{Im}[\chi] \langle \hat{\sigma}_z \rangle = \frac{-2g^2 \Gamma_{\text{tot}} \langle \hat{\sigma}_z \rangle}{\Gamma_{\text{tot}}^2 + \Delta^2} \tag{12}
\end{align}

where we have introduced the full linewidth $\Gamma_{\text{tot}} = \frac{\gamma}{2} + \Gamma_\phi$ and the detuning between qubit and resonator $\Delta = \omega_d - \omega_r$.

**Supplementary References**


