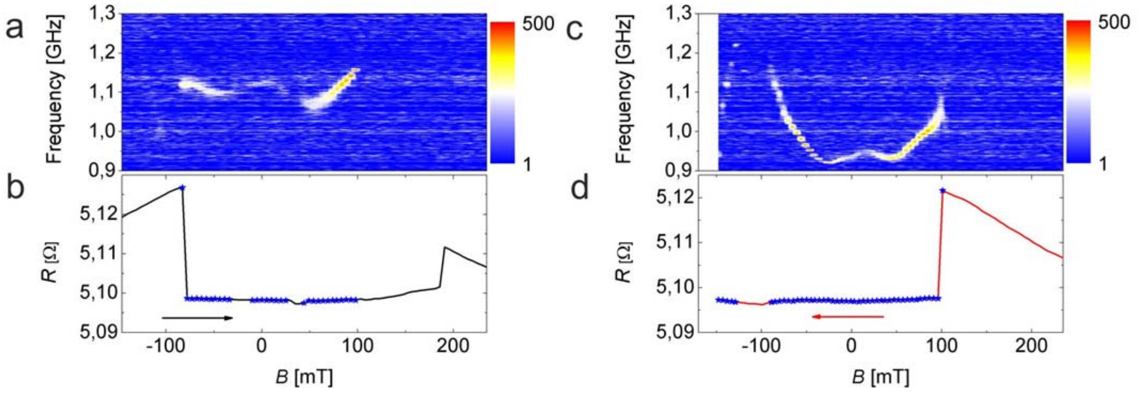
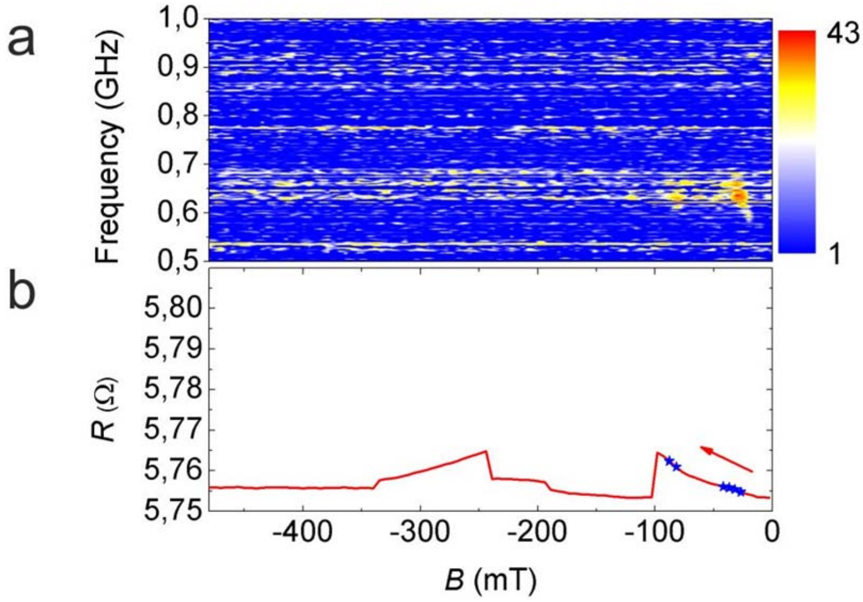
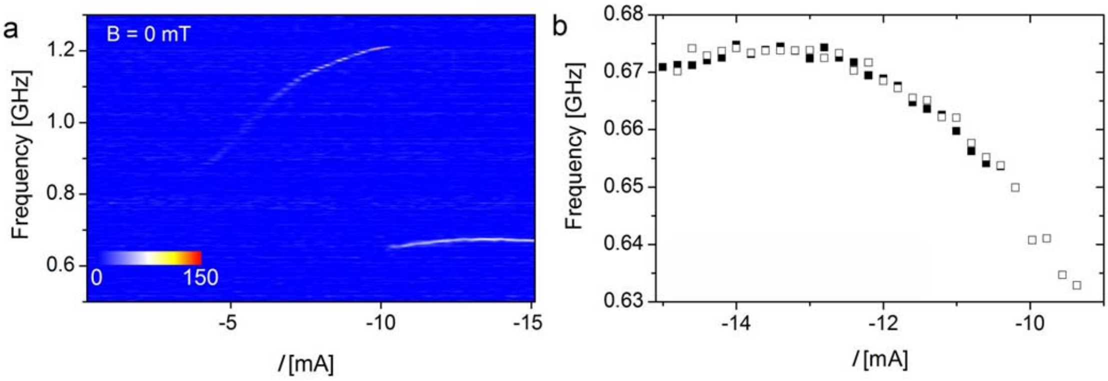


Supplementary Figure 1: Magnetoresistance and frequency splitting of configurations with equal vorticities. The figure shows combined dc and high-frequency magnetoresistance measurements as a function of the applied field *B* for EV-configurations with both vorticities parallel to the Oersted field. Panels (a) and (c) display the high-frequency data obtained during the corresponding measurements of the dc resistance *R* shown in panels (b) and (d), respectively. The arrows in panel (b) and (d) refer to the direction of the field sweeps. The electron flow during these measurements is from the top to the bottom electrode. The nucleation of the EV states are accompanied by a downwards jump of the resistance as can be seen in panels (b) and (d). The blue stars on the resistance curve indicate the presence of peaks in the simultaneously measured spectra [c.f. panels (a) and (c)]. The bar shows the spectral power density measured in pW GHz^{-1} (logarithmic scale). The two modes displayed by panels (a) and (c) clearly differ from each other: Comparing their frequencies at low fields yields values of about 1.12 GHz (panel (a)) and 0.95 GHz (panel (c)). Since the two EV states exhibit identical vorticity combinations (the same as the Oersted field), these results show that the relative core polarities differ for the two states and that EV-modes are split in a similar way compared to the OV-states.



Supplementary Figure 2: EV-state with vorticities opposite to the Oersted field. The figure displays high-frequency spectra (panel (a)) and dc resistance *R* (panel (b)), of an EV state with vorticities opposite to that of the Oersted field, measured as a function of the magnetic field *B*. The arrow in panel (b) indicates the field sweep direction. The electron flow in this experiment is directed from the top to the bottom electrode. Excitations are clearly visible at low fields around 40 and 80 mT (the bar shows the logarithmic spectral power density in pW GHz^{-1}). At fields above 100 mT, the magnetic configuration becomes unstable as both vortices switch their vorticity to align with the Oersted field. A second mode with the same Oersted field/ vorticity combination but the alternative relative core alignment is expected to exist, but was not observed in the experiment. A possible reason could be that the minimum current to excite that mode is higher than the maximum current applicable without reversing the vorticities of the EV-state.



Supplementary Figure 3: Measurement of the 0.64 GHz-mode. In some cases, after performing the state preparation procedure for $b_{CW}t_{CCW}$ –type states, a mode was observed that, according to the simulations, does not correspond to any of the OV-DV states. This mode is characterized by a high dc magnetoresistance similar to the OV-DV states and a frequency of about 1.2 GHz close to -10 mA. Panel (a) shows the frequency of this mode measured as a function of the applied current *I* in zero field, the bar referring to the spectral power density in pW GHz^{-1} (logarithmic scale). Around -10 mA, the mode becomes unstable and the frequency exhibits a drop of several hundred MHz. This drop in frequency is accompanied by a 7 mΩ upwards step in the dc resistance (not shown). After the switch, the frequency values are between 0.65 and 0.68 GHz, consistent with the numerical results for vortex gyration in $b_{CW}t_{CCW}$ –type states. After reaching -15 mA, the current is swept back to zero. A comparison of the current-dependent frequencies for both sweep directions is shown in panel (b). Black symbols correspond to the sweep to high negative current, white squares refer to the subsequent sweep back towards zero current. The measurement data ascertains the absence of any significant hysteretic behavior, i.e. the mode shows a reversible frequency-versus-current profile, as one may expect for the case of a DV state. This allows to extract the frequency of this mode close to -10 mA, in order to compare it to the other OV-DV modes. The frequency, measured at zero field, is 0.64 GHz. Its value is the lowest of all four frequencies, indicating that the corresponding state is $b^{\uparrow}_{CW}t^{\uparrow}_{CCW}$ or $b^{\downarrow}_{CW}t^{\downarrow}_{CCW}$, i.e. the core polarities are parallel to each other.

Supplementary Note 1: Coupled vortex dynamics of EV-states as function of the applied external field

Double vortex states of equal vorticities can be prepared in a way analogous to the preparation of OV states. Similar to the OV-configurations, a fine-splitting is expected to occur between the EV-states, too. The measurement data shown in Supplementary Figs 1-2 display indeed three different modes: In case of the two modes shown in Supplementary Fig. 1, both vorticities are parallel to the circulation of the Oersted field, indicating that the frequency difference at low fields arises from different relative core alignments. In the measurement displayed in Supplementary Fig. 2, Oersted field and vorticities were opposite to each other. The data shows that the configuration can be excited with a dc current, and the relatively low frequency value (around 640 MHz) can be explained by a reduction of the effective field confining the vortex, since the Oersted-field is locally antiparallel to the magnetization in each layer. We have not detected a fourth mode, although it could be expected, as it would correspond to the alternative relative core alignment. The low signal to noise ratio in Supplementary Fig. 3 suggests that the missing mode might just require a larger current to be excited in a detectable manner, which would however destabilize the configuration, eventually leading to a vorticity switching. In all three cases displayed in Supplementary Figs. 1 and 2, a positive current of 15 mA was used to excite the coupled system, corresponding to electrons flowing from the top to the bottom disc.