Supplementary Information

dc voltages in (Ga,Mn)As structures induced by ferromagnetic resonance

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Supplementary Figure S1 | Measurement configurations for a single (Ga,Mn)As layer on undoped GaAs. (a) Sample is rotated about [001] orientation in order to investigate in-plane magnetic anisotropy. The definitions of in-plane magnetic field angle $\varphi_H$ and magnetization angle $\varphi_M$ are presented. (b) Sample is rotated about [110] orientation. The dc voltage is also collected during ferromagnetic resonance measurements. The definitions of out-of-plane magnetic field angle $\theta_H$ and magnetization angle $\theta_M$ are presented. We use two Cartesian coordinate systems $(x, y, z)$ and $(x', y', z')$ in analyses.
Supplementary Figure S2 | Magnetic-field angle dependence of resonant fields and linewidth for (Ga,Mn)As/undoped GaAs. (a) In-plane field angle $\varphi_H$ dependence of resonant fields $H_R$ (open circles) at 45 K. Solid line is a fitted line by equation (S4). Dashed line is a fitted line with an additional uniaxial easy axis along [100] orientation. (b) Out-of-plane field angle $\theta_H$ dependence of $H_R$ (open symbols) at 45 K. Solid lines is a fitted line by equation (S5). (c) Out-of-plane field angle $\theta_H$ dependence of linewidths $\Delta H$ (open symbols) at 45 K. Solid line is a calculated line by using $\chi''$ in equation (S9). Inset shows $\theta_H$ dependence of magnetization angle $\theta_M$. 

$\mu_0 H_B = 45.0$ mT

$\mu_0 H_U = 30.0$ mT

$g = 1.989$

$\mu_0 H_{UB} = 4.0$ mT

$\mu_0 H_K = 413.7$ mT

$\alpha = 0.026$
Supplementary Figure S3 | Calculated magnetic-field angle dependence of susceptibility. (a) $\chi \sin \theta_M$, (b) $\chi'' \sin \theta_M$, (c) $\chi'_a \sin \theta_M$, and (d) $\chi''_a \sin \theta_M$, all are normalized by microwave absorption coefficient $I$. 
Supplementary Figure S4 | Ferromagnetic resonance spectra and dc voltage curves of (Ga,Mn)As/undoped GaAs. (a) Ferromagnetic resonance (FMR) (derivative absorption spectra of (Ga,Mn)As on undoped GaAs at 45 K as a function of out-of-plane field angle $\theta_H$. (b) dc voltage measured simultaneously during FMR measurements, where offset voltage $V_{offset}$ is subtracted.
Supplementary Figure S5 | Magnetic-field angle dependence of $V_{a\text{-sym}}$ and fitting by adopting phase shift $\Phi$ as a fitting parameter. Out-of-plane field angle $\theta_H$ dependence of $V_{a\text{-sym}}$ normalized by microwave absorption coefficient $I$ at 45 K. Solid line is a fitting line by using equation (S18).

The inset shows an enlarged part of the standard deviation as a function of $\Phi$, which shows a minimum at $\Phi \sim 90^\circ$. 
Supplementary Figure S6 | Planar Hall resistance and anomalous Hall resistance of (Ga,Mn)As measured by dc transport method. (a) In-plane magnetic field $\varphi_H$ dependence of planar Hall resistance $R_{\text{PHE}}$ of (Ga,Mn)As measured at 45 K with a constant in-plane magnetic field of 50 mT. Solid line is calculated $R_{\text{PHE}}$ by using anisotropy magnetic fields obtained from ferromagnetic resonance (FMR). (b) The perpendicular magnetic field $H_\perp$ dependence of anomalous Hall resistance $R_{\text{AHE}}$ at 45 K. Solid line is calculated $R_{\text{AHE}}$ by using anisotropy magnetic field obtained from FMR. (c) Temperature dependence of saturation values $R_{\text{AHE}0}$ of $R_{\text{AHE}}$. 
Supplementary Figure S7 | Ferromagnetic resonance and dc voltage of (Ga,Mn)As/undoped GaAs when sample is rotated about [001] orientation. (a) Ferromagnetic resonance spectrum and dc voltage at magnetic field angle $\phi_H = -45^\circ$, (b) $\phi_H = 0^\circ$ and (c) $\phi_H = 45^\circ$. 
Supplementary Note 1: Analyses of ferromagnetic resonance spectra

As reference measurements, we measure the ferromagnetic resonance (FMR) spectra (derivative absorption spectra of microwave obtained by sweeping an external magnetic field $H$ with small imposed ac field $H_{ac}$ (1 mT, 100 kHz)) of a 20-nm thick Ga$_{0.935}$Mn$_{0.065}$As layer grown on undoped GaAs. The two measurement configurations are adopted; one is that the sample is rotated about [001] orientation in TE$_{011}$ microwave cavity (Supplementary Figure S1a), and the other is that the sample is rotated about [110] orientation (Supplementary Figure S1b, the same configuration as that in Fig. 1a). The microwave frequency is 9.0 GHz. We form two In ohmic contacts at the two ends of the (Ga,Mn)As and measure the dc voltage $V$ between them during FMR measurements for the latter configuration.

For a (Ga,Mn)As with biaxial compressive strain grown on a GaAs (001) substrate, the magnetostatic energy density $F$ is given by

$$F = \frac{M}{2} \left\{ -2H[\cos \theta_M \cos \theta_H + \sin \theta_M \sin \theta_H \cos(\varphi_M - \varphi_H)] \right. \\
+ H_K \cos^2 \theta_M - \frac{H_B}{2} \frac{3 + \cos 4\varphi_M}{4} \sin^4 \theta_M - H_U \sin^2 \left( \varphi_M - \frac{\pi}{4} \right) \sin^2 \varphi_M \right\} \tag{S1}$$

Here $M$ is the magnetization, $H_K$ the effective perpendicular uniaxial magnetic anisotropy field including demagnetization field along [001] orientation, $H_B$ the in-plane biaxial magnetic anisotropy field along $\langle 100 \rangle$, and $H_U$ the in-plane uniaxial magnetic field along $\langle 110 \rangle$. The terms correspond in order of appearance to the Zeeman energy, effective perpendicular anisotropy energy,
in-plane cubic anisotropy energy, and in-plane uniaxial anisotropy energy. The magnetization \(M\) angles \(\phi_M\) (in-plane) and \(\theta_M\) (out-of-plane) as well as magnetic field \(H\) angles \(\phi_H\) and \(\theta_H\) are defined in Supplementary Figure S1. By imposing energy minimum conditions, \(\partial F/\partial \theta_M = 0\), \(\partial^2 F/\partial \theta_M^2 > 0\), \(\partial F/\partial \phi_M = 0\), and \(\partial^2 F/\partial \phi_M^2 > 0\), one can determine the \(M\) direction.

The resonant condition for FMR is expressed as\(^{34,36}\),

\[
\left(\frac{\omega}{\gamma}\right)^2 = \frac{\mu_0^2}{M^2 \sin^2 \theta_M} \left[ \frac{\partial^2 F}{\partial \theta_M^2} - \left( \frac{\partial^2 F}{\partial \phi_M^2} \right) \right]^2 \tag{S2}
\]

where \(\mu_0\) is the permeability in vacuum, \(\omega\) the angular frequency of magnetization precession, \(\gamma\) the gyromagnetic ratio (\(\gamma = g \mu_B / h\)), \(g\) the Landé g factor, \(\mu_B\) the Bohr magneton, and \(h\) the Dirac constant. Each term in equation (S2) can be described as

\[
\frac{\partial^2 F}{\partial \theta_M^2} = M(Ha_1 + b_1), \quad \frac{1}{\sin^2 \theta_M} \frac{\partial^2 F}{\partial \phi_M^2} = M(Ha_1 + b_2), \quad \frac{1}{\sin^2 \theta_M} \frac{\partial^2 F}{\partial \theta_M \partial \phi_M} = Mb_3
\]

with

\[
a_1 = \cos \theta_M \cos \theta_H + \sin \theta_M \sin \theta_H \cos (\phi_M - \phi_H)
\]

\[
b_1 = \left[ H_K + H_U \cos^2 (\phi_M + \frac{\pi}{4}) \right] \cos 2 \theta_M + H_B \frac{\cos 4 \theta_M - \cos 2 \theta_M}{2} \frac{3 + \cos 4 \phi_M}{4}
\]

\[
b_2 = -H_K \cos^2 \theta_M + H_B \sin^2 \theta_M \left( \cos 4 \phi_M - \cos^2 \theta_M \frac{3 + \cos 4 \phi_M}{4} \right) - H_U \left( \sin 2 \phi_M + \cos \theta_M \cos \left( \phi_M + \frac{\pi}{4} \right) \right)
\]

and

\[
b_3 = \frac{1}{2} \cos \theta_M \left( \frac{3}{2} H_B \sin 4 \phi_M \sin^2 \theta_M + H_U \cos 2 \phi_M \right)
\]

Then, the resonant condition of equation (S2) can be rewritten as
We use equation (S3) to describe the experimental data obtained for configurations (I) in Fig. S1a ($\theta_H = 90^\circ$) and (II) in Fig. S1b ($\varphi_H = 135^\circ$). We rewrite further equation (S3) of the resonant condition for the two specific configurations as follows:

(I) $\theta_H = 90^\circ$

\[
\left( \frac{\alpha}{\gamma} \right)^2 = \mu_0^2 H_1 H_2 \\
H_1 = H \cos(\varphi_H - \varphi_M) + H_K + H_B \frac{3 + \cos 4\varphi_M}{4} + H_U \sin^2 \left( \varphi_M - \frac{\pi}{4} \right) \\
H_2 = H \cos(\varphi_H - \varphi_M) + H_B \cos 4\varphi_M - H_U \cos \left( 2\varphi_M - \frac{\pi}{2} \right)
\]

(II) $\varphi_H = 135^\circ$

\[
\left( \frac{\alpha}{\gamma} \right)^2 = \mu_0^2 H_1 H_2 \\
H_1 = H \cos(\theta_H - \theta_M) + \left( -H_K - H_U - \frac{H_B}{4} \right) \cos 2\theta_M + \frac{H_B}{4} \cos 4\theta_M \\
H_2 = H \cos(\theta_H - \theta_M) + \left( -H_K - H_U + \frac{H_B}{2} \right) \cos^2 \theta_M + \frac{H_B}{2} \cos^4 \theta_M - H_B + H_U
\]

The obtained magnetic spectra are fitted by the derivative of the symmetric Lorentz function $L_{\text{sym}} = \Delta H^2/[4(H-H_R)^2 + \Delta H^2]$ multiplied by $-2I/\pi \Delta H$, $-(2I/\pi \Delta H) dL_{\text{sym}}/dH = -(16I/\pi) \Delta H(H-H_R)/[4(H-H_R)^2 + H^2]^2$, where $I$ is the absorption coefficient, to obtain the resonant fields $H_R$ and linewidths $\Delta H$ (full width at half maximum: FWHM) of the spectra. Open symbols in Supplementary Figure S2 summarize the $H$ angle dependence of $H_R$ at temperature $T = 45$ K and microwave power $P = 40$ mW, where Supplementary Figure S2a is for configuration (I) with $\theta_H = 90^\circ$ and Supplementary Figure S2b for configuration (II) with $\varphi_H = 135^\circ$. Solid lines in Supplementary Figures S2a and S2b present the fitted lines by equations (S4) and (S5), respectively.
which can reproduce the experimental data well. The obtained fitting parameters are $\mu_0 H_K = 417.3$ mT, $\mu_0 H_B = 45.0$ mT, $\mu_0 H_U = 30.0$ mT, and $g = 1.989$, which are reasonable values for typical (Ga,Mn)As layers$^{36,43}$. As seen in Supplementary Figure S2a, in-plane [010] and [100] are not equivalent orientations, indicating the presence of an additional uniaxial anisotropy field $H_{U2}$ along [100] orientation, whose existence was confirmed by transport measurements$^{44}$. The dashed line in Supplementary Figure S2a shows the fitting line taking into account the presence of $\mu_0 H_{U2} = 4$ mT. Since the magnitude of $H_{U2}$ is much smaller than other anisotropy fields, we neglect its contribution to reduce complexity in the present analyses. The inset in Supplementary Figure S2b shows $\theta_H$ as a function of $\theta_H$ determined from (S1) with $\varphi_M = 135^\circ$ and parameters given in Supplementary Figures S2a and S2b.

Symbols in Supplementary Figure S2c shows the $\theta_H$ dependence of $\mu_0 \Delta H$ for configuration (II). In order to describe the experimental results, we adopt the Landau-Lifshitz-Gilbert (LLG) equation$^{39}$

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mu_0 \mathbf{H}_{\text{eff}} + \frac{\alpha}{|\mathbf{M}|} \mathbf{M} \times \frac{d\mathbf{M}}{dt} \quad \text{(S6)}$$

where $\mathbf{M} = (m_x e^{i\omega t}, m_y e^{i\omega t}, M_z)$ is the magnetization vector, $d\mathbf{M}/dt$ its time derivative, $\mathbf{H}_{\text{eff}}$ the effective magnetic field vector, which is the sum of anisotropy magnetic field vectors, their dynamic component vectors, an external magnetic field vector, and microwave field vector $\mathbf{h} = e^{i\omega_0 t}(h, 0, 0)$ with angular frequency $\omega_0$, and $\alpha$ the damping constant. Here, we use a Cartesian coordinate system with $x$ along [110] and $z$ along the precession axis of $\mathbf{M}$ (Supplementary Figure S1b). The first term in right-hand side in equation (S6) expresses the field-torque inducing $M$ precession, and thus from equation (S6), by neglecting the second term, one can derive the resonant conditions shown above. The second term is the damping torque resulting in the relaxation of $M$, which determines the FMR
linewidth when inhomogeneous effects can be neglected. In order to obtain the dynamical magnetic complex susceptibility

\[
\chi_{\text{dyn}} = \left( \begin{array}{cc} \chi & -i\chi_a \\ i\chi_a & \chi \end{array} \right), \quad \chi = \chi' - i\chi'', \quad \chi_a = \chi_a' - i\chi_a''
\] (S7)

we extract the dynamic component of magnetization, \(m_x\) and \(m_y\), from (S6)

\[
i\omega \begin{pmatrix} m_x e^{i\omega t} \\ m_y e^{i\omega t} \end{pmatrix} = -\mu_0 \gamma \begin{pmatrix} H_1 m_x e^{i\omega t} \\ h \end{pmatrix} + i\alpha \omega \begin{pmatrix} -m_x e^{i\omega t} \\ m_y e^{i\omega t} \end{pmatrix}
\] (S8)

From equation (S8) with \(\omega = \omega_0\), one obtains

\[
\chi' = -\frac{H_1 (H_1^R H_2^R - H_1 H_2) M_z}{(H_1 H_2 - H_1^R H_2^R)^2 + \alpha^2 H_1^R H_2^R (H_1 + H_2)^2}
\]
\[
\chi'' = \frac{\alpha \sqrt{H_1^R H_2^R [H_1 (H_1 + H_2) + (H_1^R H_2^R - H_1 H_2)] M_z}}{(H_1 H_2 - H_1^R H_2^R)^2 + \alpha^2 H_1^R H_2^R (H_1 + H_2)^2}
\]
\[
\chi_a' = -\frac{\sqrt{H_1^R H_2^R (H_1^R H_2^R - H_1 H_2) M_z}}{(H_1 H_2 - H_1^R H_2^R)^2 + \alpha^2 H_1^R H_2^R (H_1 + H_2)^2}
\]
\[
\chi_a'' = -\frac{\alpha H_1^R H_2^R (H_1 + H_2) M_z}{(H_1 H_2 - H_1^R H_2^R)^2 + \alpha^2 H_1^R H_2^R (H_1 + H_2)^2}
\] (S9)

where \(H_1^R\) and \(H_2^R\) are \(H_1\) and \(H_2\) at \(H_R\). For \(\phi_H = 135^0\), according to equation (S5)

\[
H_1^R = H_R \cos(\theta_H - \theta_M) + \left( -H_K - H_U - \frac{H_B}{4} \right) \cos 2\theta_M + \frac{H_B}{4} \cos 4\theta_M
\]

\[
H_2^R = H_R \cos(\theta_H - \theta_M) + \left( -H_K - H_U + \frac{H_B}{2} \right) \cos^2 \theta_M + \frac{H_B}{2} \cos^4 \theta_M - H_B + H_U
\] (S10)

Since the microwave absorption is proportional to \(\chi''\), by comparing FMR spectra with \(H\) dependence of \(\chi''\) at each \(\theta_H\), one can determine the value of \(\alpha\). As shown in Supplementary Fig. S2c, by adopting \(\alpha = 0.026\), the \(\theta_H\) dependence of \(\mu_0 \Delta H\) can be reproduced well by equation (S9).

Supplementary Figure S3 shows dynamical susceptibility as a function of \(\theta_H\) calculated by equation (S9), in which one can see different \(\theta_H\) dependences of diagonal (\(\chi', \chi''\)) and off-diagonal
Supplementary Note 2: dc voltages in the vicinity of resonant fields

The dynamical magnetic susceptibility is related to the dynamical components of magnetization:

\[
\begin{pmatrix}
  \alpha_x \\
  \alpha_y
\end{pmatrix} = \begin{pmatrix}
  \chi & -i\chi_a \\
  i\chi_a & \chi
\end{pmatrix} \begin{pmatrix}
  h e^{i\phi} \\
  0
\end{pmatrix} = \begin{pmatrix}
  \chi \left| \alpha_x \right| - i\chi_a \left| \alpha_y \right| \\
  i\chi_a \left| \alpha_x \right| - \chi \left| \alpha_y \right|
\end{pmatrix} \begin{pmatrix}
  h e^{i\phi} \\
  0
\end{pmatrix} e^{-i\delta}
\]

where \(\delta\) is the phase shift of magnetization precession from microwave oscillation, which changes by 180\(^{\circ}\) around the resonance, and \(\Phi\) is the phase difference between microwave electric field \(\epsilon\) and magnetic field \(h\). In a microwave cavity, \(\Phi\) can be approximated to be 90\(^{\circ}\) because of standing wave character of microwave in cavity, and we also confirm it experimentally as shown in next section. From equation (S11), the real and imaginary parts of \(m_x\) and \(m_y\), \(\text{Re}(m_x)\), \(\text{Im}(m_x)\), \(\text{Re}(m_y)\), and \(\text{Im}(m_y)\) are obtained as

\[
\text{Re}(m_x) = \chi' h, \quad \text{Im}(m_x) = \chi'' h, \quad \text{Re}(m_y) = -\chi'_a h, \quad \text{Im}(m_y) = \chi''_a h
\]

For the (Ga,Mn)As/p-GaAs bilayer, since the pure spin current is induced by the spin-pumping effect, the dc voltage \(V_{\text{ISHE}}\) in p-GaAs layer induced by the inverse spin Hall effect (ISHE) is expected to be proportional to the time average of damping term (the second term) in equation (S6). By using equation (S12), one can derive the expression for \(V_{\text{ISHE}}\) as

\[
V_{\text{ISHE}} \propto \text{Re} \left( \frac{\alpha}{|M|} M \times \frac{dM}{dt} \right)_z, = \frac{\alpha \omega}{M_z^2} [\text{Im}(m_x) \text{Re}(m_y) - \text{Re}(m_x) \text{Im}(m_y)] \sin \theta_M
\]

\[
\propto -(\chi'_a \chi'' + \chi''_a \chi') \sin \theta_M
\]

The over-line indicates the time average of signal and the subscript \(z'\) represents \(z'\) component of the signal in the Cartesian coordinate system with \(x'\) along [110], \(y'\) along [001], and \(z'\) along [110] orientation (see Fig. 1b). Supplementary Figure S4 shows the FMR spectra (upper panels, from
which we obtain \( H_R \) and \( \Delta H \) in Supplementary Figures S2b and S2c) and \( V \) (bottom panels) of a single (Ga,Mn)As layer grown on undoped GaAs at \( T = 45 \) K and \( P = 40 \) mW as a function of \( \theta_H \) for configuration (II) (Supplementary Figure S1b). Although no spin pumping and thus no \( V_{\text{ISHE}} \) are expected, we observe characteristic \( V \) signals in the vicinity of \( H_R \), indicating that effects other than the ISHE should be considered. They are most likely to be galvanomagnetic effects (the possible contributions from thermoelectric effects are ruled out as shown below). It should be noted that the observed \( V \) in Supplementary Figure S4 includes both of symmetric (absorptive) and anti-symmetric (dispersive) components.

In TE\(_{011}\) cavity, the electric field is zero only at the centre "point" of the cavity. In this work, we use a standard cylindrical cavity with a diameter of 2.5 cm and a length of 4.4 cm. Since the sample is in millimeter size to observe FMR absorption with sufficient sensitivity, the circumferential electric field of microwave along \( z' \) direction acting on the sample produces galvanomagnetic effects\(^{25}\). A phenomenological description of electrical conduction in magnetic materials is given by\(^{25,38}\)

\[
J = \sigma E - \left( \frac{\Delta \rho}{\rho M^2} \right) (J \cdot M) M + R_S \sigma \mathbf{j} \times \mathbf{M}
\]  

(S14)

where \( J \) is the current density vector, \( E \) the electric-field vector, \( \sigma \) the electrical conductivity, \( \rho \) the resistivity, \( \Delta \rho \) the magnitude of the resistivity change due to the anisotropic magnetoresistance (AMR) effect, and \( R_S \) the anomalous Hall coefficient. The first term in right-hand side of equation (S14) corresponds to the Ohm’s law, and the second term to the AMR effect, and the third term to the anomalous Hall effect (AHE). For the present configuration with \( \Phi = 90^\circ \), we detect the AMR effect orthogonal to \( J \), which is often called planar Hall effect (PHE) despite that it is not a genuine Hall effect in the strict sense\(^{29,38}\). Note that one should consider longitudinal AMR effect either when \( J \) has a component along detection direction or \( \Phi \) is shifted from \( 90^\circ \)\(^{22,27}\). From equations (S12) and (S14), one can derive the expressions for dc voltages \( V_{\text{PHE}} \) and \( V_{\text{AHE}} \) induced by PHE and
AHE as

\[
\begin{align*}
V_{\text{PHE}} & \propto \Re[\mathbf{M} \cdot \mathbf{\varepsilon} \mathbf{M}], \propto \mathcal{E} M_x \cos \theta_M \propto \chi'' \sin \theta_M \\
V_{\text{AHE}} & \propto \Re[\mathbf{M} \times \mathbf{\varepsilon}], \propto \mathcal{E} M_z \cos \theta_M \propto -\chi_{a} \sin \theta_M
\end{align*}
\]  

(S15)

where \( \mathcal{E} \) in the magnitude of microwave electric field in the sample. From equations (S13) and (S15), one can see that the voltages induced by the PHE and AHE are determined solely by the real or imaginary part of the susceptibility. On the other hand, the voltage induced by the ISHE is determined by the combinations of the real and imaginary parts of susceptibility, and generates \( \Phi \) independent symmetric dc voltage\(^{14}\).

Figure 5 in main text shows the calculated lineshapes and \( \theta_H \) dependences of dc voltages induced by the ISHE, PHE, and AHE. Since the spin pumping effect broadens \( \Delta H \) of (Ga,Mn)As on p-GaAs from that on undoped GaAs, in calculation we adopt \( \alpha \) as an adjustable parameter to reproduce the experimentally obtained \( \Delta H \) of (Ga,Mn)As/p-GaAs at each \( \theta_H \) shown in Fig. 2f. The lineshape of \( V_{\text{AHE}} \) indicates that the observed anti-symmetric component of voltage in Figs. 6b and Supplementary Figure S4b is resulted from AHE\(^{14}\). The \( V_{\text{PHE}} \) contributes to symmetric component of voltage in Fig. 6a and Supplementary Figure S4b. The \( V_{\text{ISHE}} \) also contributes to symmetric component of voltage in Fig. 6a. The difference in \( \theta_H \) dependence of \( V_{\text{ISHE}} \) and \( V_{\text{PHE}} \) (Figs. 5d and 5e) can be used to extract the ISHE and PHE components from \( \theta_H \) dependence of \( V_{\text{sym}} \), and the result in Fig. 5f can be used to fit the \( \theta_H \) dependence of \( V_{\text{a-sym}} \). The fitting to the \( \theta_H \) dependence of \( V_{\text{sym}} \) by the calculated \( V_{\text{ISHE}} \) and \( V_{\text{PHE}} \) shows that apparent contribution from \( V_{\text{ISHE}} \) for (Ga,Mn)As/undoped GaAs is very small (~2%, the order of experimental error), while sizable contribution (~12%) from that for (Ga,Mn)As/p-GaAs.

Supplementary Note 3: Experimental determination of \( \Phi \)

Although in any ideal cavity, \( \Phi = 90^\circ \) due to standing wave character of microwave, \( \Phi \) may be
shifted from $90^\circ$ by the presence of the sample in the cavity. According to equation (S11), $\text{Re}(m_x)$ and $\text{Re}(m_y)$ can be expressed as

$$\text{Re}(m_x) = (\chi'' \sin \Phi + \chi' \cos \Phi) h, \quad \text{Re}(m_y) = (\chi_a'' \cos \Phi - \chi_a' \sin \Phi) h$$

(S16)

Thus, the expressions of $V_{\text{PHE}}$ and $V_{\text{AHE}}$ induced by PHE and AHE are

$$V_{\text{PHE}} \propto (\chi'' \sin \Phi + \chi' \cos \Phi) \sin \theta_M, \quad V_{\text{AHE}} = (\chi_a'' \cos \Phi - \chi_a' \sin \Phi) \sin \theta_M$$

(S17)

Since $\chi'$ and $\chi_a'$ have anti-symmetric lineshape and $\chi''$ and $\chi_a''$ have symmetric lineshape in their $H$ dependence, both $V_{\text{PHE}}$ and $V_{\text{AHE}}$ have symmetric and anti-symmetric contributions to $V$ if $\Phi$ is neither 0 nor $90^\circ$. The total anti-symmetric part $V_{\text{a-sym}}$ in $V$ induced by PHE and AHE ($V_{\text{PHE}}^{\text{a-sym}}$ and $V_{\text{AHE}}^{\text{a-sym}}$) can be written as

$$V_{\text{a-sym}} = V_{\text{PHE}}^{\text{a-sym}} + V_{\text{AHE}}^{\text{a-sym}} = (\chi' \cos \Phi - \chi_a' \sin \Phi) \sin \theta_M$$

(S18)

We fit $\theta_H$ dependence of $V_{\text{a-sym}}$ in Fig. 6b with equation (S18) and the calculated $\chi_{\text{dyn}}$ in Supplementary Figure S3 by adopting $\Phi$ as a fitting parameter. Supplementary Figure S5 shows the fitting results, and the inset shows the standard deviation as a function of $\Phi$ in the vicinity of $90^\circ$, which has a minimum at $\Phi \sim 90^\circ$.

**Supplementary Note 4: dc transport measurements of anomalous and planar Hall effects of (Ga,Mn)As**

The planar Hall resistance $R_{\text{PHE}}$ and anomalous Hall resistance $R_{\text{AHE}}$ of (Ga,Mn)As cleaved from the same wafer in this study are also measured by dc transport means by using Hall bar\textsuperscript{43}. Supplementary Figure S6a presents the $\phi_H$ dependence of $R_{\text{PHE}}$ under a constant in-plane $H$ of 50 mT. The result can be described well by, $R_{\text{PHE}} = R_{\text{PHE}0} \cos 2 \phi_M$, where $R_{\text{PHE}0} = 10 \ \Omega$ is the amplitude of $R_{\text{PHE}}$.\textsuperscript{49} Here, the in-plane magnetization direction $\phi_M$ is determined from magnetostatic energy
density in equation (S1) with $\theta_M = 90^\circ$ by imposing energy minimum condition $\partial F/\partial \varphi_M = 0$ and
$\partial^2 F/\partial \varphi_M^2 > 0$ and by using the in-plane magnetic anisotropy fields, $\mu_0 H_B = 45.0$ mT and $\mu_0 H_U = 30.0$ mT, determined from FMR (Supplementary Fig. S2a).

Supplementary Figure S6b shows the perpendicular $H$ dependence of $R_{AHE}$ at 45 K, which can be reproduced well by the calculated $H$ dependence (solid line) of $R_{AHE} = R_{AHE0}\cos \theta_M$ by using equation (S1) with $\varphi_M = 135^\circ$ and $\mu_0 H_K = 413.7$ mT obtained from FMR (Supplementary Figure S2b), where $R_{AHE0}$ is the saturation value of $R_{AHE}$. The $T$ dependence of $R_{AHE0}$ is shown in Supplementary Figure S6c. Nonmonotonic $T$ dependence of $R_{AHE0}$ is consistent with nonmonotonic $T$ dependence of the magnitudes of anti-symmetric component of dc voltage in Supplementary Figure 4c, which is attributed to the AHE. The dc transport measurements also support comparable magnitudes of symmetric $V_{sym}$ and anti-symmetric $V_{a-sym}$ components in dc voltages (Figs. 3c, 4b, and 4c). The ratio between the magnitudes of dc planar Hall resistance $R_{PHE0}$ and anomalous Hall resistance $R_{AHE0}$ is 1:4.6 at $T = 45$ K (Supplementary Figure S6). The LLG equation gives that the ratio of magnitude of $\text{Re}(m_x)$ and $\text{Re}(m_y)$ is 4.2:1 at $T = 45$ K and $\theta_H = 90^\circ$. Since $V_{sym}$ is dominated by the PHE, we may write $V_{sym}:V_{a-sym} \sim \text{Re}(m_x)R_{PHE0}:\text{Re}(m_y)R_{AHE0}$ [from equation (S15)], which leads to $V_{sym} \sim V_{a-sym}$.

By comparing the magnitudes of dc voltages under FMR and those obtained by dc transport measurements, we calculate the radio-frequency (rf) current density in (Ga,Mn)As in the cavity to be $\sim 10^3$ A cm$^{-2}$. This value is at least two orders smaller than the current density required to observe spin-orbit interaction driven FMR with $V \sim 100$ $\mu$V in similar (Ga,Mn)As$^{50}$.

Supplementary Note 5: Rule out of possible heating effect due to microwave
absorption

The microwave absorption at resonance may heat the magnetic layer and may create heat current, which can be the source of dc voltage through transverse thermoelectric (Nernst-Ettingshausen) effect (ordinary and/or anomalous Nernst effects)\(^5\). The heating effect in observed \(V\) for the present system can be ruled out by adopting in-plane measurement configuration for the reference sample, (Ga,Mn)As on GaAs substrate (Supplementary Figure S1a). Being independent of measurement configurations, the heat current induced by FMR is expected to have symmetric Lorentzian lineshape and its direction to be along \(y'\) direction ([001] orientation from surface to substrate).

When the in-plane \(z'\) component of \(M\) or \(H\) exists with the heat current, the anomalous or ordinary Nernst effects can produce dc voltage with symmetric Lorentzian lineshape in (Ga,Mn)As. Supplementary Figure S7 presents the FMR spectra and \(V\) measured at \(T = 80\) K and \(P = 30\) mW at three \(\phi_H\)'s, in which no characteristic \(V\) is observed around \(H_R\), indicating that the heating effect due to microwave absorption can be neglected. The result also shows that the configuration dependent electric field distribution in the sample sometimes helps to eliminate the magnetogalvanic effects (however, not always as shown in Ref. 14).

**Supplementary Note 6: Mixing conductance and spin Hall angle**

The real part of mixing conductance \(g^{\uparrow\downarrow}_r\) at the interface of (Ga,Mn)As and p-GaAs is expressed as\(^{22,52,53}\)

\[
g^{\uparrow\downarrow}_r = \frac{2\sqrt{3} \pi M_S d_{\text{GMA}}}{g \mu_B \omega} (\Delta H_{\text{GMA/P}} - \Delta H_{\text{GMA/I}}) \tag{S19}
\]

where \(M_S\) is the saturation magnetization of (Ga,Mn)As, \(d_{\text{GMA}}\) is the thickness of (Ga,Mn)As, and \(\Delta H_{\text{GMA/P}}\) and \(\Delta H_{\text{GMA/I}}\) are the linewidths of the ferromagnetic resonance spectra for (Ga,Mn)As on p-GaAs and on undoped GaAs, respectively. By substituting \(M_S \sim 60\) mT, \(d = 20\) nm, and
\( \mu_0(\Delta H_{GMA/P} - \Delta H_{GMA/I}) = 2.17 \) mT into equation (S19), \( g_r^{\uparrow\downarrow} = 4.74 \times 10^{18} \) m\(^{-2}\). This is about an order smaller than those determined for ferromagnetic metal systems, which is due to smaller \( M_S \) of (Ga,Mn)As than that of standard metal magnets.

According to the theory of spin pumping, the dc voltage induced by the ISHE is described as \(^{53}\)

\[
V_{ISHE} = \frac{2e\theta_{SHE}}{h} \left( \sigma_{pGA} + \frac{d_{GMA}}{d_{pGA}} \sigma_{GMA} \right)^{-1} \lambda_{pGA} \times \tanh \left( \frac{d_{pGA}}{2\lambda_{pGA}} \right) J_s^0
\]

(S20)

where \( e \) is the elementary charge, \( l \) the length of the device, \( \theta_{SHE} \) the spin Hall angle, \( \sigma_{pGA} \) the conductivity of p-GaAs, \( d_{pGA} \) the thickness of p-GaAs, \( \lambda_{pGA} \) the spin diffusion length of p-GaAs, and \( \sigma_{GMA} \) the conductivity of (Ga,Mn)As. The pure spin current density is given by

\[
J_s^0 = \frac{g_r^{\uparrow\downarrow} \gamma^2 h^2 \left( 4\pi M_S \gamma + \sqrt{(4\pi M_S \gamma)^2 + 4\omega^2} \right)}{8\pi^2 \left[ (4\pi M_S \gamma)^2 + 4\omega^2 \right]}
\]

(S21)

By using \( V_{ISHE} = 1.77 \mu V \), \( l = 2 \) mm, \( \sigma_{pGA} = 1 \times 10^4 \) \( \Omega^{-1} m^{-1} \), \( \sigma_{GMA} = 2 \times 10^4 \) \( \Omega^{-1} m^{-1} \), \( d_{pGA} = 20 \) nm, \( \lambda_{pGA} = 5.6 \) nm\(^{20}\), and \( h = 27 \) \( \mu T \), the spin Hall angle is calculated to be \( \theta_{SHE} = 0.006 \). Up to now, there is no report on \( \theta_{SHE} \) in p-GaAs, this value is comparable to that in n-GaAs\(^{54-56}\), although the ISHE in p-GaAs and n-GaAs may have different origins\(^8,57\). If the entire symmetric component \( (V_{sym} = 15.0 \mu V) \) is treated as the ISHE signal, the spin Hall angle is calculated to be \( \theta_{SHE} = 0.05 \), which is comparable with heavy metals [Ref. 41 and references therein]. Such a large spin Hall angle suggests that the assumption, that \( V_{sym} \) is dominated by the ISHE, is not correct.

In the present (Ga,Mn)As/p-GaAs, the conductivities of (Ga,Mn)As is comparable to that of p-GaAs, because of two-order larger hole concentration despite two-order lower mobility\(^{58}\). Thus, rf current density in p-GaAs is \( \sim 10^3 \) A cm\(^{-2}\) similar to that in (Ga,Mn)As as shown above. Such small current density and relatively small spin Hall angle in p-GaAs are obviously too small to excite FMR and thus to observe spin-torque induced spin rectification effect\(^{41,59}\).
Supplementary References


56. Garlid, E. S. *et al.* Electrically measurement of the direct spin Hall effect in Fe/In$_x$Ga$_{1-x}$As

