Supplementary Information

Supplementary Discussion of Dark Domain Length Analysis

The magnetic interactions should produce even-length domains of dark sites, corresponding to AF spin domains. To quantify the length of these AF domains we study the length of the measurable dark domains, defining a “dark domain” as a contiguous string of dark sites that is bounded either by a site with an atom or an edge of the region of interest. We then calculate the mean length-weighted dark chain length from this data.

Defects in the initial Mott insulator (MI) reduce the effective system size. Their appearance can produce an overestimate of the dark chain length by either connecting two dark chains, or appearing on the end of a dark chain. The initial MI defect probability is typically 4% per site over an entire N=1 shell, after correcting for losses during imaging.

Losses and higher order tunneling processes during the ramp can have similar consequences for the observed dark domain length, and can also suppress the observed dark domain length by perturbing atoms near the end of the ramp once the AF has already formed. The rate of such processes can be estimated from the MI 1/e squeezing lifetime in the tilted lattice, measured to be 3.3 seconds. To perform this measurement we first ramp to a tilt of 300Hz/lattice site and tune the lattice depths to 45Er and 14Er for transverse and longitudinal lattices, respectively. We then hold for a variable time, and measure the observed in situ atom number. The lifetime is dominated by higher-order tunneling processes. Parametric heating and inelastic scattering become the dominant loss channel in deeper lattices once tunneling is inhibited.

A worst-case estimate for the impact of missing atoms can be reached from the fraction of the time that the system is missing no atoms at the end of the ramp. The six-site chain analyzed in the main text is initially fully occupied 79% of the time. During the time it takes the dark-domain length to grow to 4 lattice sites (60 ms), the aforementioned effects only reduce this number to 73%.

Supplementary Table 1 shows the entropy per particle \( S/Nk_B \) for several different Mott insulator fidelities \( p_{\text{ood}} \), assuming a chemical potential \( \mu = U / 2 \), as well as the mean length-weighted AF domain size \( D \) in an infinite 1D magnetic system with the same entropy per particle. Here \( D = 2(2 - \epsilon) / \epsilon \), where the spin-dislocation probability in the AF \( \epsilon \) is defined by \( S/Nk_B \).
Spin defects are ignored as they are both dynamically and thermodynamically unlikely. The entropy per particle can be related to the Mott insulator fidelity by

\[ S/Nk_B = \log\left[ \frac{2}{1-p_{\text{odd}}} \right] - p_{\text{odd}} \log\left[ \frac{2p_{\text{odd}}}{1-p_{\text{odd}}} \right]. \]

If such thermalization took place in our finite length chain of 6-sites (with initial fidelity 97.5%), the mean domain size would be limited by the system size to 5.3 sites.

<table>
<thead>
<tr>
<th>Mott Fidelity (p_{\text{odd}})</th>
<th>Entropy per Particle (S/Nk_B)</th>
<th>Length-Weighted Mean AF Domain Size (Thermalized)</th>
<th>Length-Weighted Mean Uninterrupted Chain Length (Unthermalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0.23</td>
<td>22</td>
<td>39</td>
</tr>
<tr>
<td>0.975</td>
<td>0.13</td>
<td>52</td>
<td>79</td>
</tr>
<tr>
<td>0.99</td>
<td>0.063</td>
<td>144</td>
<td>199</td>
</tr>
</tbody>
</table>

**Supplementary Table 1:** For various Mott insulator fidelities, the corresponding configurational entropy per particle is computed. These entropies are comparable to, or well below, the critical entropy for quantum magnetism \(S/Nk_B\approx0.25-0.5\). If the spin degrees of freedom thermalize efficiently with the Mott degrees of freedom, the spin entropy will then be equal to the Mott entropy. The corresponding mean AF domain size is then computed for each Mott entropy. In the absence of thermalization, the Mott defects break the spin chain into disconnected subsystems, whose mean size is computed in the fourth column, and is comparable to the mean chain length in the presence of thermalization.

Experimentally, we find most Mott defects to be unbound doublons and holes, which do not directly map to excitations in the spin model. The large energy gap present in our tilted lattice, combined with conservation of particle number, make it difficult for these Mott defects to thermalize with spin degrees of freedom. Such thermalization would require, for example, migration of a doublon to a hole, or decay via a very high order process into several spin defects- quite unlikely within the experimental timescale. Consequently, these nearly static defects act as fixed boundary conditions that limit the effective length of the simulated spin chains. Supplementary Table one also provides the expected uninterrupted chain length, computed as \(L_{\text{sys}}=(1+p_{\text{odd}})/(1-p_{\text{odd}})\).

**Impact of Selecting a Sub-System with Low Disorder**

As described above and in the main text, tilt inhomogeneity limits our system-size to 6 sites. Ideally one would decouple the 6-site chain from neighboring high-disorder regions by projecting a sharp potential barrier at each end of the chain, before the magnetic sweep.

Because we have not performed such a decoupling procedure, our 6-site chain is a sub-system of a larger chain. The boundary conditions of the larger chain can then be pinned by a combination of MI defects, and disorder, typically within 1~2 sites beyond the ends of the 6-site chain.
Supplementary Discussion of Higher Order Effects

Interchain Tunneling:

Tunneling between chains is excluded from the spin-mapping described in the main text, though under certain conditions it produces exotic transverse superfluidity, as described in Ref. [4]. For our purposes, these tunneling processes serve only to take the system out of the Hilbert space described by the spin model. Most of our experiments were performed at a transverse lattice depth of 45Er, corresponding to an interchain tunneling rate of $t_{\text{transverse}} = 2\pi \times 0.07$ Hz. This tunneling rate is basically negligible on our experiment timescale of 250 ms. The noise correlation data, as well as the shell pictures and reversibility curve in Fig. 2 of the main text, were taken at 35Er transverse lattice depth. At this depth the transverse tunneling rate is $t_{\text{transverse}} = 2\pi \times 0.27$ Hz, which is small compared to our lattice inhomogeneities, and so results in highly-suppressed, off-resonant Rabi-flopping. In practice, increasing the transverse lattice from 35Er to 45Er results in a modest ~5% improvement in the quality of the Mott insulator after transitioning to the antiferromagnetic state and back.

Second Order Tunneling:

In addition to nearest-neighbor tunneling which creates doublon-hole pairs, and proceeds at a rate $\sqrt{2t}$ when the tilt $E=U$, there remain second-order tunneling processes which create triplons at a rate $t_{\text{s.o.}} \sim \frac{\sqrt{3}t^2}{U}$. For our longitudinal lattice depth of 14 Er, and interaction energy $U = 2\pi \times 416$ Hz, we find $t_{\text{s.o.}} \sim 2\pi \times 0.4$ Hz.

Because our system is continuously tilted, all such transitions will be tuned through resonance. For our typical experiment, $R_{\text{ramp}} = \frac{1}{2} \frac{U}{250 \text{ms}} = 2\pi \times 840 \text{Hz}^2$, so the Landau-Zener adiabatic transition probability to the triplon state $P_{\text{triplon}} = 1 - \exp \left( -2\pi \frac{t_{\text{s.o.}}^2}{R_{\text{ramp}}} \right) \sim 1\%$. In future experiments with slower ramps, both this effect and the closely related second-order Stark-shift will become more of a concern. These can be further suppressed relative to the desired dynamics by increasing the longitudinal lattice depth, at the expense of slower many-body dynamics. It bears mentioning that for our experimental parameters the triplon state should experience an additional energy shift calculated to be 22 Hz, due to multi-orbital interactions$^{5,6}$.

Impact of physics beyond the Hubbard Model

For a 14Er lattice, the next-nearest neighbor tunneling rate is suppressed relative to that of the nearest neighbor$^7$ by a factor of ~300, making the total rate $t_{\text{NextNeighbor}} = 2\pi \times 0.04$ Hz, which is negligible on
present experiment timescales. The longitudinal nearest-neighbor interaction shift for one atom per lattice site is \( \sim 10^{-3} \) Hz, and interaction driven tunneling\(^8\) occurs with a rate of \( 2\pi \times 0.3 \) Hz.

**Supplementary Discussion of Exact Diagonalization Parameters**

Figure 4 of the main text contains exact diagonalizations of a few-body spin Hamiltonian in panes b and d. For 4.b, harmonic confinement is simulated via a longitudinal field gradient of 0.01 per lattice site. This diagonalization is for a chain of 7 spins with open boundary conditions, as harmonic confinement breaks the translational symmetry and makes periodic boundary conditions meaningless. The transverse field that drives the transition is given by \( h_x=0.001 \). For 4.d, the harmonic confinement is removed, and a chain of 6 spins is exactly diagonalized with periodic boundary conditions. The number of spins was here chosen to be even because the system experiences frustration and degenerate ground states for odd chain lengths and periodic boundary conditions.

Supplementary Figure 1: Single-site transition curve: The occupation probability \( p_{\text{odd}} \) of a characteristic single site, plotted versus tilt as the system is ramped from the PM phase into the AF phase. The theory curve reflects a zero temperature exact diagonalization calculation of the ground state of a chain of six Ising spins (the shape of the \( p_{\text{odd}} \) curve is insensitive to chain length, see SI Fig. 3), with periodic boundary conditions. The curve has been offset and rescaled vertically to account for defects arising from both the initial MI, and heating during the ramp. The theory allows us to extract a lattice depth of 14(1)Er, in agreement with 15(2)Er measured by Kapitza-Dirac scattering. We attribute the residual fluctuations around the expected curve to residual oscillations.
reflecting non-adiabaticity arising from that fact that the ramp was initiated too close to the transition. The error bars are 1 $\sigma$ statistical uncertainties.

Supplementary Figure 2: Modulation spectroscopy in a tilted lattice: The occupation probability averaged over the 6-site near-homogeneous region described in the main text, plotted versus the modulation frequency, for 16Er longitudinal lattice modulated by ±23%, corresponding to a Bose-enhanced resonant tunneling rate of $2\pi \times 4$ Hz. Because the experiment is performed in a lattice tilted by $E$ per site, the peak at zero tilt which appears the interaction energy $U$ is split out into two peaks, one corresponding to an atom tunneling up the tilt at an energy cost of $U+E$, and one to tunneling down the tilt a cost of $U-E$. Fitting these peaks allows us to extract both $U$, and $E$. The peak width arises from a combination of power broadening (approximately $2\pi \times 14$ Hz, complicated by Rabi flopping), and the residual lattice disorder discussed in the main text.
**Supplementary Figure 3: Comparing $S_z$ to the order parameter:** An exact diagonalization calculation (for $h_z=.004$) of the ground state of a 1D chain of 4 (black) and 8 (red) Ising spins with nearest neighbor interactions, revealing that $S_z$ (solid) is not sensitive to atom number, while the order parameter (dash-dotted) is. It is anticipated that the order parameter will exhibit a cusp in the large-system limit, though the exponential scaling with atom number precludes simulating substantially larger systems on a classical computer.

**Bibliography**


5. Will, S. *et al.*, Time-resolved observation of coherent multi-body interactions in quantum


**Supplementary Information** is linked to the online version of the paper at www.nature.com/nature.