An Open-System Quantum Simulator with Trapped Ions

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I. BELL-STATE PUMPING

A. Implemented Kraus maps

The Bell state $|\Psi^-(1)\rangle$ is not only uniquely determined as the simultaneous eigenstate with eigenvalue -1 of the two stabilizer operators $X_1X_2$ and $Z_1Z_2$ (as mentioned in the text), but also by $X_1X_2$ and $Y_1Y_2$. In the experiment, we implemented pumping into $|\Psi^-(1)\rangle$ by engineering the two Kraus maps $\rho_S \mapsto E_1\rho_S E_1^\dagger + E_2\rho_S E_2^\dagger$ and $\rho_S \mapsto E_1'\rho_S E_1'^\dagger + E_2'\rho_S E_2'^\dagger$, where

$$E_1 = \sqrt{p} Y_1 \frac{1}{2} (1 + X_1X_2), \quad E_2 = \frac{1}{2} (1 - X_1X_2) + \sqrt{1 - p} \frac{1}{2} (1 + X_1X_2) \quad (1)$$

$$E_1' = \sqrt{p} X_1 \frac{1}{2} (1 + Y_1Y_2), \quad E_2' = \frac{1}{2} (1 - Y_1Y_2) + \sqrt{1 - p} \frac{1}{2} (1 + Y_1Y_2), \quad (4)$$

which generate pumping into the -1 eigenspaces of $X_1X_2$ and $Y_1Y_2$ (instead of pumping into the eigenspaces of $X_1X_2$ and $Z_1Z_2$ as explained in Box 1 of the main text). The reason for pumping into the eigenspaces of $X_1X_2$ and $Y_1Y_2$ is that the mapping and unmapping steps, shown as (i) and (iii) in Box 1, are realized by a single MS gate $U_{X^2}(\pi/2)$ and $U_{Y^2}(\pi/2)$, respectively.

B. Circuit decomposition

The map for pumping into the -1 eigenspace of $X_1X_2$ can be realized by the unitary

$$U_{X^2(\pi/2)} C(p) U_{X^2(\pi/2)} \quad (5)$$

(corresponding to steps (i) - (iii) in Box 1) followed by optical pumping of the ancilla qubit to $|1\rangle$. Here, the two-qubit controlled gate is

$$C(p) = |0\rangle\langle 0| \otimes \exp(iaZ_1) + |1\rangle\langle 1| \otimes \mathbb{1}$$

$$= \exp \left[ \frac{1}{2} (1 + Z_0)iaZ_1 \right]$$

$$= U_{Z_1}(\alpha) U_Y(\pi/2) U_{X^0_1}^{(0,1)}(\alpha) U_Y(\pi/2) \quad (6)$$

where $U_{X^0_1}^{(0,1)}(\alpha) = \exp(i\alpha/2)X_0X_1$ denotes an MS gate acting only on the ancilla and the first system qubit. This two-qubit MS gate operation was implemented in the experiment by the use of refocusing techniques (Nebendahl et al., 2009). In more detail, the gate $U_{X^0_1}^{(0,1)}$ was realized by interspersing two of the available three-qubit MS gate operations with single-ion light
shifts on the second system qubit which induces a $\pi$-phase shift between the qubit states. Alternatively, this refo-
cusing could be avoided, and the sequences further sim-
plicated, by hiding the population of individual ions (here
the second system ion) which are not supposed to partic-
ipate in collective coherent operations in electronic levels
decoupled from the driving laser excitation. More details
on how to systematically decompose Kraus maps into the
experimentally available ion-trap gate operations, in par-
ticular the multi-ion MS entangling gate, can be found
in Müller \textit{et al.} (2010).

The circuit decompositions for the actual experimental
implementation of the two maps are shown in Fig. S1.
They differ from the two quantum operations, which
are specified in Eqs. (1)-(4), by two single-ion rotations.
They arise since the circuit has been slightly modified (by
changing the phase of one of the global $Y$-rotations) at
the expense of implementing in addition in each dissip-
itive map a flip operation $Y_1Y_2$ on the two system qubits.
However, as this additional unitary corresponds to one
of the stabilizers into whose -1 eigenspace the pumping
is performed, this does not interfere with the pumping
dynamics.

Pumping with unit pumping probability $p = 1$ corre-
sponds to $\alpha = \pi/2$, whereas $p = 0.5$ is realized by setting
$\alpha = \pi/4$. In the experiment, the ”fundamental” MS gate
was calibrated to implement $U_{X2}(\alpha/2)$. The fully entan-
gling operation $U_{X2}^2(\alpha/2)$ at the beginning and the end of
the sequence Fig. S1a was then implemented by applying
the $U_{X2}(\alpha/2)$ operation twice (for $p = 1$) or four times
(for $p = 0.5$). The fully entangling operations $U_{Y2}(\pi/2)$
in Fig. S1b were implemented by two- and four-fold ap-
plication of the “fundamental” MS gate with a shifted
optical phase of the driving laser (cf. Section 2 in the
main text).

**FIG. S1** Experimental sequences for Bell-state pump-
ing. Pumping into the eigenspaces of eigenvalue -1 of $X_1X_2$
(circuit a) and $Y_1Y_2$ (circuit b) occurs with a probability $p$
in each step, where $\sin^2 \alpha = p$. The circuit is up to two local
rotations equivalent to the quantum operations specified in
Eqs. (1)-(4).

### C. Towards master equation dynamics

For an implementation of pumping dynamics with small pumping probabilities $p \ll 1$, described by a multi-
qubit master equation with two-qubit quantum jump op-
erators, several requirements have to be met:

From a practical point of view, to reach the desired
target Bell state via pumping with small pumping prob-
abilities $p$ requires increased gate fidelities as more time
steps are needed to come close to the steady state of the
repeated pumping dynamics. This implies that pro-
cesses, such as e.g. decay of population into decoupled
electronic states, which correspond to “leakage” out of
the logical Hilbert space, have to be kept small after a
larger number of gate operations. More fundamentally,
even in the absence of these leakage processes, the errors
in the implementation of the dissipative maps eventually
hinder the system from coming arbitrarily close to the
target state. Loosely speaking, one can think of the ideal
dissipative dynamics as describing an infinite set of non-
reversible paths along which any initial state is pumped
towards the desired target state. Deviations from
the ideal path during the preparation due to implementation
errors and other disturbances then place the system onto
points in Hilbert space, which differ from the ideally ex-
pected ones. From there, the system “gets a new chance”
and is again attracted towards the target state under sub-
sequent dissipative operations. However, as the system
approaches the vicinity of the target state, the errors
hinder the system from coming closer and closer to the
target state. Here, a balance between pumping towards
and repulsion from the target state builds up, which is
closely related to the error level and the chosen pumping
probability: here the pumping rate $p$ for populating the
target state competes with the loss processes at a rate $\epsilon$
due to implementation errors. This competition leads to
a steady state infidelity scaling as $\propto \epsilon/p$.

In the experiment on Bell state pumping into $|\Psi^-\rangle$
at a pumping rate $p = 0.5$, we have carried out up to
three and a half pumping cycles and observed that the
two system qubits reached a maximum overlap fidelity
with the target Bell state of 73(1)% after three pumping
cycles.

### D. Further experimental details

As mentioned in the main text, fully mixed states
of two and four qubits were prepared by a dissipative
process based on optical pumping. First, every sys-
tem qubit, initially prepared in $|0\rangle$, is coherently trans-
ferred to $(|0\rangle + |S\rangle)/\sqrt{2}$ via a $\pi/2$ laser pulse on the
quadrupole transition, where $|S\rangle$ is the electronic level
$S_{1/2}(m = 1/2)$. Subsequently, optical pumping of the
population in $|S\rangle$ into $|1\rangle$ creates a state where the co-
herence between the resulting populations in $|0\rangle$ and $|1\rangle$

is completely destroyed.

The initial two-qubit mixed state was prepared with a fidelity of $F = 99.6(3)\%$ with respect to the ideal state $\frac{1}{4}^{1}\rangle_{4\times4}$.

Physical process matrices were reconstructed with maximum likelihood techniques (Jezek et al., 2003). An error analysis was carried out via Monte Carlo simulations over the multinomially distributed measurement outcomes of the state and process tomography. For each process and state, 200 Monte Carlo samples were generated and reconstructed via maximum-likelihood estimation.

II. FOUR-QUBIT STABILIZER PUMPING

Expectation values of the stabilizer operators $Z_1Z_2$, $Z_2Z_3$, $Z_3Z_4$ and $X_1X_2X_3X_4$ were not determined from the reconstructed density matrices of the system qubits. Instead, we performed fluorescence measurements in the $X$ and $Z$ basis on 5250 copies of the corresponding quantum states (for $p = 0.5$ pumping, 2100 copies were measured). The error bars were then determined from the multinomially distributed raw data.

A. Pumping

Pumping into the GHZ state $(|0000\rangle + |1111\rangle)/\sqrt{2}$ was realized by a pumping cycle where the four system qubits were deterministically pumped into the $+1$ eigenspaces of the stabilizers $Z_1Z_2$, $Z_2Z_3$, $Z_3Z_4$ and $X_1X_2X_3X_4$.

The ideal dissipative Kraus map describing the first three pumping steps into the $+1$ eigenspace of $Z_1Z_2$, $Z_2Z_3$ and $Z_3Z_4$ read $\rho_S \mapsto E_{iZ}^{id} = E_1\rho S E_1^\dagger + E_2\rho S E_2^\dagger$ with

$$E_1 = \frac{1}{2}(1 + Z_iZ_j),$$

$$E_2 = \frac{1}{2}X_j(1 - Z_iZ_j),$$

for $(i, j = 12, 23, 34)$. The Kraus maps are constructed such that the $+1$ eigenspace of $Z_iZ_j$ is left invariant, whereas a spin flip $X_j$ on the second spin (index $j$) converts with unit probability -1 into +1 eigensates.

The dissipative map for pumping into the $+1$ eigenspace of, e.g., $Z_1Z_2$ could be achieved in complete analogy with Bell state pumping, i.e. by effectively only implementing operations on the ancilla qubit and the system qubits #1 and #2, whereas the system qubits #3 and #4 remain completely unaffected. This could either be achieved through refocusing techniques or by hiding system ions #3 and #4 in electronically decoupled states for the duration of the dissipative circuit.

In the experiment, however, we used a few simplifications that allowed us to simplify the employed circuits.

These are schematically shown in Fig. S2 and listed below:

- For deterministic pumping ($p = 1$), the inverse mapping step (shown in Box 1) is not necessary and has been taken out.
- In the coherent mapping step (shown in Box 1) the information about whether the system ions are in a $\pm 1$ eigenstate of $Z_1Z_2$ is mapped onto the logical states of the ancilla qubit. This step ideally only involves the ancilla and the system qubits #1 and #2. One way to achieve this three-qubit operation without affecting the system qubits #3 and #4, is to combine the available five-ion MS gate with appropriately chosen refocusing pulses, i.e. light shift operations on individual ions. Those would have to be chosen such that ions #0, #1 and #2 become decoupled from ions #3 and #4, and furthermore residual interactions between ions #3 and #4 cancel out. However, it turns out that residual interactions between ions #3 and #4 can be tolerated: although not required for the $Z_1Z_2$-pumping dy-
themics, they are not harmful, as they do not alter the expectation values of the other two-qubit stabilizers $Z_2Z_3$ and $Z_3Z_4$. In our experiment the decoupling of ions #0, #1 and #2 from the ions #3 and #4 was achieved by the circuit shown in Fig. S2b.

The additional interactions in the pumping of the two-qubit stabilizer operators $Z_iZ_j$ affect the state of the system qubits with respect to the four-qubit stabilizer $X_1X_2X_3X_4$. However, this effect is not detrimental to the pumping, provided the pumping into the eigenspace of $X_1X_2X_3X_4$ is performed as the final step in the pumping cycle.

- In the employed sequence, the number of single-qubit rotations was reduced wherever possible. Essential single-qubit light shift operations, as those needed for re-focusing operations, were kept.

- Local rotations of the system ions at the end of a pumping step, which would be compensated at the beginning of the subsequent pumping step, were omitted when several dissipative maps were applied in a row. The corresponding gate operations of the sequences are displayed in blue in Steps 1-3.

These simplifications allowed us to significantly reduce the length and complexity of the employed gate sequences for one stabilizer pumping step. As a consequence, the actual Kraus map for pumping into the $+1$ eigenspace of the stabilizer operator $Z_1Z_2$ as implemented in the experiment is $\rho_{S} \rightarrow \mathcal{E}_{Z_1Z_2}^{exp}(\rho_{S}) = E_1\rho_{S}E_1^{\dagger} + E_2\rho_{S}E_2^{\dagger}$ with

$$E_1 = X_1X_2 \left( \hat{A} - Z_1\hat{S} \right) X_2 \frac{1}{2} (1 - Z_1Z_1),$$

$$E_2 = \left( X_1X_2\hat{S} + \frac{i}{2} (X_1Y_2 + Y_1X_2) \hat{A} \right) \frac{1}{2} X_2 (1 - Z_1Z_2),$$

where

$$\hat{S} = X_3X_4 + Y_3Y_4 \text{ and } \hat{A} = X_3Y_4 + Y_3X_4.$$  

This quantum operation differs from the ideal Kraus map specified in Eqs. (7) and (8) by combinations of additional simultaneous $X$ and $Y$-type spin flips on all four system spins. As explained above, these additional terms do not interfere with the $Z_iZ_j$ pumping dynamics, as any four-spin operator, which is built up by a product of either $X$ or $Y$ for each of the four spins commutes with the $Z$-type two-body stabilizers, e.g. $[X_1X_2Y_3Y_4, Z_iZ_j] = 0$.

The experimental Kraus maps for pumping into the $+1$ eigenspaces of $Z_2Z_3$ and $Z_3Z_4$ are obtained from Eqs. (9)-(11) by applying the corresponding permutation of system spin indices.

The fourth dissipative step, which realizes pumping into the $+1$ eigenspace of $X_1X_2X_3X_4$, is described by the ideal and also experimentally implemented Kraus map

$$\rho_{S} \rightarrow \mathcal{E}_{X_1X_2X_3X_4}^{exp}(\rho_{S}) = E_1\rho_{S}E_1^{\dagger} + E_2\rho_{S}E_2^{\dagger}$$

with

$$E_1 = \frac{1}{2} \left( 1 + X_1X_2X_3X_4 \right),$$

$$E_2 = \frac{1}{2} Z_4 (1 - X_1X_2X_3X_4).$$

The gate sequences, which have been used in the experiment to implement these Kraus maps are explicitly given below:

Step 1 (pumping into the $+1$ eigenspace of $Z_1Z_2$):

$$U_Y(-\pi/2)U_{Z_2}(-\pi/2)$$

$$U_X(\pi/2)U_{Z_3}(-\pi/2)U_X(-\pi/2)$$

$$U_{Z_1}(\pi)U_{XZ_2}(\pi/4)U_{Z_2}(\pi)U_{Z_3}(\pi)U_{XZ_2}(\pi/4)$$

$$U_X(-\pi/2)U_{Z_2}(-\pi/2)U_{Z_3}(\pi/2)U_X(\pi/2)$$

$$U_{Z_2}(\pi/4)U_{Z_1}(\pi)U_{Z_2}(\pi)U_{XZ_2}(\pi/4)$$

$$U_Y(\pi/2)U_X(-\pi/2)U_{Z_3}(\pi/2)U_X(\pi/2)$$

Step 2 (pumping into the $+1$ eigenspace of $Z_2Z_3$):

$$U_Y(-\pi/2)U_{Z_3}(-\pi/2)$$

$$U_X(\pi/2)U_{Z_2}(-\pi/2)U_X(-\pi/2)$$

$$U_{Z_1}(\pi)U_{Z_2}(\pi/4)U_{Z_2}(\pi)U_{Z_3}(\pi)U_{XZ_2}(\pi/4)$$

$$U_X(-\pi/2)U_{Z_2}(-\pi/2)U_{Z_3}(\pi/2)U_X(\pi/2)$$

$$U_{Z_2}(\pi/4)U_{Z_1}(\pi)U_{Z_2}(\pi)U_{XZ_2}(\pi/4)$$

$$U_Y(\pi/2)U_X(-\pi/2)U_{Z_3}(\pi/2)U_X(\pi/2)$$

Step 3 (pumping into the $+1$ eigenspace of $Z_3Z_4$):

$$U_Y(-\pi/2)U_{Z_4}(-\pi/2)$$

$$U_X(\pi/2)U_{Z_3}(-\pi/2)U_X(-\pi/2)$$

$$U_{Z_1}(\pi)U_{Z_3}(\pi/4)U_{Z_3}(\pi)U_{Z_4}(\pi)U_{XZ_3}(\pi/4)$$

$$U_X(-\pi/2)U_{Z_3}(-\pi/2)U_{Z_4}(\pi/2)U_X(\pi/2)$$

$$U_{Z_3}(\pi/4)U_{Z_1}(\pi)U_{Z_3}(\pi)U_{XZ_3}(\pi/4)$$

$$U_Y(\pi/2)U_X(-\pi/2)U_{Z_4}(\pi/2)U_X(\pi/2)$$

Step 4 (pumping into the $+1$ eigenspace of $X_1X_2X_3X_4$):

$$U_X(-\pi/2)$$

$$U_{Z_4}(-\pi/2)U_X(\pi/2)U_{Z_4}(-\pi/2)$$

$$U_{Z_1}(\pi/4)U_{Z_4}(\pi)U_{Z_4}(\pi)U_{XZ_4}(\pi/4)$$

$$U_X(-\pi/2)U_{Z_4}(-\pi/2)U_{Z_4}(\pi/2)U_X(\pi/2)$$

$$U_{Z_4}(\pi/4)U_{Z_1}(\pi)U_{Z_4}(\pi)U_{XZ_4}(\pi/4)$$

Figure S10 shows the reconstructed density matrices (real and imaginary parts) for every step of the pumping cycle. The complete circuit decomposition of one pumping cycle involves 16 five-ion entangling operations, 28 (20) collective unitaries and 36 (34) single-qubit operations with (without) optional operations in blue. The reset operation involves further pulses not accounted for above.
B. Repeated four-qubit stabilizer pumping

To study the robustness of the dissipative operation, we prepared the initial state $|1111\rangle$ and subsequently applied repeatedly the dissipative map for pumping into the +1 eigenspace of the four-qubit stabilizer $X_1X_2X_3X_4$. We observed that after a single dissipative step a non-zero expectation value of $X_1X_2X_3X_4$ built up and stayed constant under subsequent applications of this dissipative map. However, due to imperfections in the gate operations, the expectation values of the two-qubit stabilizers decreased, ideally they should not be affected by the $X_1X_2X_3X_4$-pumping step (see Fig. S3). Interestingly, the expectation values of $Z_1Z_4$ and $Z_3Z_4$ decayed significantly faster than those for $Z_1Z_2$ and $Z_2Z_3$. This decay can be explained by the fact that in the gate sequence used for pumping into the +1 eigenspace of $X_1X_2X_3X_4$, step 4 above, single-ion light-shift operations are applied only to the fourth system qubit and the ancilla. This indicates that errors in the single-qubit gates applied to the fourth system ion accumulate under the repeated application of the dissipative step, and thus affect the stabilizers $Z_1Z_4$ and $Z_3Z_4$ which involve this system qubit more strongly than the others. This destructive effect can be minimized by alternating the roles of the system qubits.

Such optimization has been done for the dissipative dynamics shown in Fig. S4. Here, starting from the initial state $|1111\rangle$, repeated pumping into the -1 eigenspace of $X_1X_2X_3X_4$ has been implemented by the sequence

$U_{X^2}(\pi/8) U_{X^2}(\pi/8) U_{X^2}(\pi/8) U_{X^2}(\pi/8)
U_X(-\pi/2)
U_{Z^4}(-\pi/2 \times p) U_X(\pi/2) U_{Z^4}(\pi)
U_{Y^2}(\pi/4 \times p) U_{Z^o}(\pi) U_{Z^4}(\pi) U_{Y^2}(\pi/4 \times p)
U_Y(\pi/2) U_{Z^4}(-\pi/2) U_Y(-\pi/2)
U_{X^2}(\pi/8) U_{X^2}(\pi/8) U_{X^2}(\pi/8) U_{X^2}(\pi/8)$.

FIG. S3 Measured expectation value of stabilizers for repeated pumping without sequence optimization. The expectation values of $Z_1Z_4$ and $Z_3Z_4$ show a significantly faster decay than those for $Z_1Z_2$ and $Z_2Z_3$. In every step of the pumping, most single-ion light-shift operations are applied to the fourth system qubit.

FIG. S4 Measured expectation value of stabilizers for repeated pumping with sequence optimization. All two-qubit stabilizers decay at the same rate during pumping. In step 1, 2, 3, 4, and 5 the single-qubit light-shift operations were applied on the system qubits 4, 3, 2, 1, and 1, respectively. Here, we observed that indeed the expectation values of all two-qubit stabilizers decreased at the same pace and at a slightly slower rate (see Fig. S4). Upon repeating the sequence above 1, 2, 3, 4, and 5 times, we changed the operations shown in red to act on qubits 4, 3, 2, 1, and 1, respectively. The stabilizer expectation values for deterministic pumping, or $p = 1$, are shown in Fig. S4.

C. Pushing “anyons” around

In Kitaev’s toric code (Kitaev, 2003), spins are located on the edges of a two-dimensional square lattice. The Hamiltonian

$$H = -g(\sum_p A_p + \sum_v B_v)$$

is a sum of mutually commuting four-qubit stabilizers $A_p = \prod_{i \in p} X_i$ and $B_v = \prod_{i \in v} Z_i$, which describe four-spin interactions between spins located around plaquettes $p$ and vertices $v$ of the lattice. The ground state of the Hamiltonian is the simultaneous +1 eigenstate of all stabilizer operators. The model supports two types of excitations that obey anyonic statistics under exchange (braiding), and they correspond to -1 eigenstates of either plaquette or vertex stabilizers.

For a minimal instance of this model, represented by a single plaquette of four spins located on the edges, the Hamiltonian contains a single four-qubit interaction term $X_1X_2X_3X_4$ and pairwise two-spin interactions $Z_iZ_j$ of spins sharing a corner of the plaquette. The ground state as the simultaneous +1 eigenstate of these stabilizers is the GHZ-state $(|0000\rangle + |1111\rangle)/\sqrt{2}$. States corresponding to -1 eigenvalues of a two-qubit stabilizer $Z_iZ_j$ can be interpreted as a configuration with an excitation located at the corner between the two spins $i$ and $j$. Similarly, a four-qubit state with an eigenvalue of -1 with respect to $X_1X_2X_3X_4$, would correspond to an anyonic excitation located at the center of the plaquette.
In the experiment we prepared an initial state \( |0111\rangle \) and then performed the pumping cycle of four deterministic pumping steps into the +1 eigenspaces of \( Z_1Z_2, Z_2Z_3, Z_3Z_4 \) and \( X_1X_2X_3X_4 \), using the sequences for Steps 1 to 4 given in section II.A. The expectation values of the stabilizer operators for the initial state and the four spins after each pumping step are shown in Fig. S5. The dissipative dynamics can be visualized as follows: For the initial state with \( \langle Z_1Z_2 \rangle = -1 \) and \( \langle Z_1Z_4 \rangle = -1 \) a pair of excitations is located on the upper left and right corners of the plaquette, whereas \( \langle X_1X_2X_3X_4 \rangle = 0 \) implies anyon of the other type is present at the center of the plaquette with a probability 50%. In the first pumping step, where the first two spins are pumped into the +1 eigenspace of \( Z_1Z_2 \), the anyon at the upper right corner is dissipatively pushed to the lower right corner of the plaquette. In the third step of pumping into the +1 eigenspace of \( Z_2Z_4 \), the two excitations located on the upper and lower left corners fuse and disappear from the system. In the final step of pumping into the +1 eigenspace of \( X_1X_2X_3X_4 \), the anyon with a probability of 50% at the center of the plaquette is pushed out from the plaquette.

However, we’d like to stress that borrowing concepts from topological spin models, such as anyonic excitations, here is merely a convenient language to phrase and visualize the dissipative dynamics. In the present work with up to five ions, we do not explore the physics of topological spin models, since (i) in a minimal system of four spins the concepts developed for larger lattice models become questionable, and more importantly, (ii) during the implemented pumping dynamics the underlying (four-body) Hamiltonian of the model was not present. We rather demonstrate the basic tools which will allow one to explore this physics once larger, two-dimensional systems become available in the laboratory.

We note that photon experiments have reported the observation of correlations compatible with the manipulations of “anyons” in a setup representing two plaquettes (Lu et al., 2009; Pachos et al., 2009). Such experiments are based on postselection of measurements (as in teleportation by Bouwmeester et al., 1997), which should be contrasted to our deterministic implementation of open system dynamics to prepare and manipulate the corresponding quantum state (as in deterministic teleportation by Barrett et al., 2004; Riebe et al., 2004).

### D. Pumping into “excited” states

Starting from an initially fully mixed state of four qubits, we also implemented pumping into a different GHZ-type state, \((|0010\rangle - |1101\rangle)/\sqrt{2}\), by a sequence of four dissipative steps: 1) pumping into the +1 eigenspace of \( Z_1Z_2 \), 2) pumping into the -1 eigenspace of \( Z_2Z_3 \), 3) pumping into the -1 eigenspace of \( Z_3Z_4 \) and 4) pumping into the -1 eigenspace of \( Z_1Z_4 \). In the context of Kitaev’s toric code, this state would correspond to an excited state. However, as above, we point out that the underlying Hamiltonian was not implemented in the pumping dynamics.

The measured expectation values of the stabilizers are shown in Fig. S6. The final density matrix, as determined from quantum state tomography after the four pumping steps, is shown in Fig. S7. This pumping cycle was implemented with the same sequences as given for Step 1 to 4 in section II.A, with the only difference that the sign of the phase shift operations displayed in red was changed in Steps 2, 3, and 4. This allowed us to invert the pumping direction from the +1 into -1 eigenspaces of \( Z_2Z_3 \), \( Z_3Z_4 \) and \( X_1X_2X_3X_4 \).

### III. QND MEASUREMENT OF A FOUR-QUBIT STABILIZER

#### A. Further details

As shown in Fig. S8, the QND measurement involves a mapping step where the information about whether the system described by an input density matrix \( \rho^{\text{in}} \) is in...
the $+1 / -1$ eigenspace of $A = X_1X_2X_3X_4$ is coherently mapped onto the internal states $|0\rangle$ and $|1\rangle$ of the ancilla qubit, which is initially prepared in $|1\rangle$. Subsequently the ancilla qubit is measured in its computational basis, leaving the system qubits in a corresponding output state $\rho_{\text{out}}$.

The coherent mapping $M(X_1X_2X_3X_4)$ was realized by the sequence

$$
U_X(\pi/4)U_{Z_0}(\pi)U_X(-\pi/4)U_{X_2}(\pi/4)U_{Z_2}(-\pi/2)U_{X_2}(\pi/4)U_{Z_2}(\pi)U_Y(-\pi/4)U_{Z_2}(\pi/4)
$$

which implements

$$
M(X_1X_2X_3X_4) = -\frac{i}{\sqrt{2}}(X_0 + Y_0) \otimes P_+ + \frac{1}{\sqrt{2}}(1 - iZ_0) \otimes P_-, \quad (15)
$$

with $P_{\pm} = \frac{1}{2}(1 \pm X_1X_2X_3X_4)$ the projectors onto the $\pm 1$ eigenspaces of $X_1X_2X_3X_4$. Equation (15) shows that for the system qubits being in a state belonging to the $+1$ eigenspace of the stabilizer operator, the ancilla is flipped from $|1\rangle$ to $|0\rangle$, whereas it remains in its initial state $|1\rangle$ otherwise.

Subsequently, the ancilla as well as the four system qubits were measured. This was done by measuring the five ions simultaneously. Alternatively, we first hid the four system qubits in electronic levels decoupled from the laser excitation, performed the fluorescence measurement of the ancilla qubit, then recovered the state of the system qubits and tomographically measured the state of the four system qubits. The second approach, where the state of the system is not affected by the measurement of the ancilla, is of importance if the information from the ancilla measurement is to be used for feedback operations on the state of the system.

**B. Quantitative analysis of the performance**

To characterize the performance of a QND measurement for a (multi-)qubit system, a set of requirements and corresponding fidelity measures have been discussed in the literature (Ralph et al., 2006).

(1) First of all, the measurement outcomes for the ancilla qubit should agree with those that one would expect from a direct measurement of the observable $A$ on the input density matrix. This property can be quantified by the measurement fidelity,

$$
F_M = \left( \sqrt{p_{\text{in}}^{m^0}} + \sqrt{p_{\text{in}}^{m^1}} \right)^2 , \quad (16)
$$

which measures the correlations of the distribution of measurement outcomes $p^m = \{p^m_0, p^m_1\}$ of the ancilla qubit with the expected distribution $p^{m^\pm} = \{p^{m^0}, p^{m^1}\}$ directly obtained from $\rho^{\text{in}}$, where $p^{m^\pm} = \text{Tr}\{\frac{1}{2}(1 \pm A)p^{m^\pm}\}$.

(2) The QND character, reflected by the fact that the observable $A$ to be measured should not be disturbed by the measurement itself, becomes manifest in ideally identical probability distributions $p^{\text{in}}$ and $p^{\text{out}}$, which are determined from the input and output density matrices. These correlations are quantified by the QND fidelity

$$
F_{\text{QND}} = \left( \sqrt{p_{\text{in}}^{\text{out}}p_{\text{in}}^{\text{out}}} + \sqrt{p_{\text{in}}^{\text{in}}p_{\text{in}}^{\text{out}}} \right)^2 , \quad (17)
$$

where $p_{\text{out}}^{\text{out}} = \text{Tr}\{\frac{1}{2}(1 \pm A)p^{\text{out}}\}$.

(3) Finally, by measuring the ancilla qubit the system qubits should be projected onto the corresponding eigenspace of the measured observable $A$. Thus the quality of the QND measurement as a quantum state preparation (QSP) device is determined by the correlations between the ancilla measurement outcomes and the corresponding system output density matrices. It can be described by the QSP fidelity

$$
F_{\text{QSP}} = p_{\text{in}}^{\pm}p_{\text{out}}^{\pm^0} + p_{\text{in}}^{\pm}p_{\text{out}}^{\pm^1} , \quad (18)
$$
where $p_{\text{out}}^{(0/1),+}$ denotes the conditional probability of finding the system qubits in the +1 (-1) eigenspace of $A$, provided the ancilla qubit has been previously measured in $|0\rangle$ ($|1\rangle$).

The probability distributions for the system input and output states, the ancilla measurement outcome distributions, and the resulting fidelity values are summarized in Tables I to IV. The input states had a fidelity (Jozsa, 1994) with the ideal states $(|0000\rangle + |1111\rangle)/\sqrt{2}$, $(|0000\rangle - |1111\rangle)/\sqrt{2}$ and $(|0011\rangle - |1100\rangle)/\sqrt{2}$ of 75.3(9), 77.3(8), 93.2(4)%.

We observe that we obtain higher values for the measurement and QND fidelities than for the QSP fidelities. The latter is relevant in the context of quantum error correction or closed-loop simulation protocols or more generally whenever the information from the ancilla measurement is used for further processing of the system output state.

With the additional hiding and unhiding pulses before and after the measurement of the ancilla we observe a loss of fidelity of a few percent in the QSP fidelities.
TABLE I QND probability distributions. Obtained from measurements with hiding of the system ions during the measurement of the ancilla.

<table>
<thead>
<tr>
<th>input state</th>
<th>eigenspace</th>
<th>( p_{\text{in}} )</th>
<th>( p_{\text{out}} )</th>
<th>( p_{\text{out}}^{m=0} )</th>
<th>( p_{\text{out}}^{m=1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>000\rangle +</td>
<td>111\rangle)</td>
<td>+1</td>
<td>0.959(1)</td>
<td>0.847(3)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.041(1)</td>
<td>0.153(3)</td>
<td>0.189(9)</td>
<td>0.178(9)</td>
</tr>
<tr>
<td>(</td>
<td>000\rangle −</td>
<td>111\rangle)</td>
<td>+1</td>
<td>0.955(1)</td>
<td>0.169(3)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.045(1)</td>
<td>0.831(3)</td>
<td>0.809(10)</td>
<td>0.813(9)</td>
</tr>
</tbody>
</table>

TABLE II QND probability distributions. Obtained from measurements without hiding of the system ions during the measurement of the ancilla.

<table>
<thead>
<tr>
<th>input state</th>
<th>eigenspace</th>
<th>( p_{\text{in}} )</th>
<th>( p_{\text{out}} )</th>
<th>( p_{\text{out}}^{m=0} )</th>
<th>( p_{\text{out}}^{m=1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>000\rangle +</td>
<td>111\rangle)</td>
<td>+1</td>
<td>0.850(3)</td>
<td>0.713(11)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.150(3)</td>
<td>0.287(11)</td>
<td>0.211(11)</td>
<td>0.664(30)</td>
</tr>
<tr>
<td>(</td>
<td>000\rangle −</td>
<td>111\rangle)</td>
<td>+1</td>
<td>0.188(3)</td>
<td>0.265(12)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.812(3)</td>
<td>0.735(12)</td>
<td>0.496(28)</td>
<td>0.780(11)</td>
</tr>
<tr>
<td>(</td>
<td>001\rangle −</td>
<td>110\rangle)</td>
<td>+1</td>
<td>0.099(2)</td>
<td>0.073(7)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.901(2)</td>
<td>0.927(7)</td>
<td>0.584(35)</td>
<td>0.962(5)</td>
</tr>
</tbody>
</table>

TABLE III QND figures of merit. Determined from measurements with hiding of the system ions during the measurement of the ancilla. Since the state \(|001\rangle − |110\rangle\) is particularly robust against decoherence, the fidelity \(F_{\text{QSP}}\) is higher, as shown for 8 ions in Monz et al. (2010).

<table>
<thead>
<tr>
<th>input state</th>
<th>eigenspace</th>
<th>( p_{\text{in}} )</th>
<th>( p_{\text{out}} )</th>
<th>( p_{\text{out}}^{m=0} )</th>
<th>( p_{\text{out}}^{m=1} )</th>
<th>( F_{\text{M}}(p_{\text{in}},p_{m}) )</th>
<th>( F_{\text{QND}}(p_{\text{in}},p_{out}) )</th>
<th>( F_{\text{QSP}}(p_{m},p_{\text{out}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>000\rangle +</td>
<td>111\rangle)</td>
<td>+1</td>
<td>0.82(1)</td>
<td>0.69(1)</td>
<td>0.85(1)</td>
<td>0.74(1)</td>
<td>0.998(1)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.18(1)</td>
<td>0.31(1)</td>
<td>0.15(1)</td>
<td>0.64(3)</td>
<td>0.999(1)</td>
<td>0.980(5)</td>
<td>0.74(1)</td>
</tr>
<tr>
<td>(</td>
<td>000\rangle −</td>
<td>111\rangle)</td>
<td>+1</td>
<td>0.041(4)</td>
<td>0.14(1)</td>
<td>0.10(1)</td>
<td>0.48(4)</td>
<td>0.985(3)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.959(4)</td>
<td>0.86(1)</td>
<td>0.90(1)</td>
<td>0.90(1)</td>
<td>0.9992(6)</td>
<td>0.73(1)</td>
<td></td>
</tr>
</tbody>
</table>

REFERENCES


TABLE IV. QND figures of merit. Determined from measurements \textbf{without} hiding of the system ions during the measurement of the ancilla. Since the state \(|0011\rangle - |1100\rangle\) is particularly robust against decoherence, the fidelity \(F_{\text{QSP}}\) is higher, as shown for 8 ions in Monz \textit{et al.} (2010).

<table>
<thead>
<tr>
<th>input state</th>
<th>eigenspace</th>
<th>(p_{\text{out}})</th>
<th>(p_{\text{in}})</th>
<th>(p_{\text{QND}=+})</th>
<th>(p_{\text{QND}=-})</th>
<th>(F_M(p_{\text{in}}, p_{\text{m}}))</th>
<th>(F_{\text{QND}}(p_{\text{in}}, p_{\text{out}}))</th>
<th>(F_{\text{QSP}}(p_{\text{m}}, p_{\text{QND}}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>0000\rangle +</td>
<td>1111\rangle)</td>
<td>+1</td>
<td>0.71(1)</td>
<td>0.85</td>
<td>0.79(1)</td>
<td>0.998(1)</td>
<td>0.984(4)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.29(1)</td>
<td>0.15</td>
<td>0.66(3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(</td>
<td>0000\rangle -</td>
<td>1111\rangle)</td>
<td>+1</td>
<td>0.26(1)</td>
<td>0.19</td>
<td>0.50(3)</td>
<td>1.0000(1)</td>
<td>0.992(3)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.74(1)</td>
<td>0.81</td>
<td>0.78(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(</td>
<td>0011\rangle -</td>
<td>1100\rangle)</td>
<td>+1</td>
<td>0.07(1)</td>
<td>0.10</td>
<td>0.42(3)</td>
<td>0.986(2)</td>
<td>0.996(2)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.93(1)</td>
<td>0.90</td>
<td>0.96(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(</td>
<td>1111\rangle)</td>
<td>+1</td>
<td>0.52(1)</td>
<td>0.5078</td>
<td>0.75(1)</td>
<td>0.99994</td>
<td>0.9996(5)</td>
<td>0.74(1)</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>0.48(1)</td>
<td>0.4922</td>
<td>0.73(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG. S9 Reconstructed process matrices of experimental Bell-state pumping. The reconstructed process matrix for $p = 1$ after 1 (1.5) cycles has a Jamiolkowski process fidelity Gilchrist et al. (2005) of 83.4(7)% (87.0(7)%) with the ideal dissipative process $\rho_S \mapsto |\Psi^-\rangle\langle \Psi^-|$ which maps an arbitrary state of the system into the Bell state $|\Psi^-\rangle$. This ideal process has as non-zero elements only the four transparent bars shown. The reconstructed process matrix for $p = 0.5$ after 3 cycles has a Jamiolkowski process fidelity of 60(1)% with the ideal process $\chi_{\text{ideal}}$ shown [$\text{Im}(\chi_{\text{ideal}}) = 0$].
FIG. S10  **Ideal and reconstructed density matrices of plaquette pumping.** An initial mixed state $\rho_{\text{mixed}}$ is sequentially pumped by the stabilizers $Z_1Z_2$, $Z_2Z_3$, $Z_3Z_4$ and $X_1X_2X_3X_4$ driving the system into the states $\rho_{1,2,3,4}$.

FIG. S11  **Pushing “anyons”**. Cartoon of the dissipative dynamics. The pumping dynamics can be visualized by dissipative pushing of excitations (green and red dots) between adjacent corners of the plaquette.