News & views

Statistical mechanics

An exceptional view of phase transitions

Cynthia J. O. Reichhardt & Charles Reichhardt

Phase transitions in certain non-equilibrium systems cannot be described using the classical laws of statistical mechanics. A mathematical approach involving features called exceptional points now solves this far-reaching problem. See p.363

Newton's third law, which states that every action has an equal and opposite reaction, is such a foundational element of classical mechanics that it is often taken for granted. But an increasing number of studies involve non-equilibrium systems in which the equality of action and reaction is broken, leading to non-reciprocal interactions between the constituent elements of the system[1]. The standard machinery of classical statistical mechanics is incapable of describing the phase transitions that occur in such systems. On page 363, Fruchart *et al.*[2] report that a mathematical model involving exceptional points – locations in the parameter space of a system at which two or more modes of the system coalesce into a single mode – successfully captures the behaviour of non-reciprocal phase transitions. The findings pave the way for the future analysis and eventual harnessing of these phase transitions in a variety of applications.

What is a non-reciprocal interaction, and how can it arise? In its simplest form, a non-reciprocal interaction between elements A and B is one in which A does not have the same effect on B as B has on A. Such interactions are impossible if all forces in the system are conservative – that is, if the total work done by the forces is independent of the path taken, a condition that guarantees the overall conservation of energy. Non-reciprocal interactions are therefore associated with either a gain or a loss of energy.

Strongly non-reciprocal interactions and non-conservative forces can arise in social interactions, such as those that occur when pedestrians avoid each other, or when birds fly together as a flock[3]. These systems are examples of 'active matter', in which each element (a person or bird, in our examples) contains an internal energy source that injects energy into the system by enabling each element to move under its own propulsion[4]. By contrast, ordinary non-active matter is purely passive, such as a leaf drifting on a river in response to the underlying currents.

Let's consider the example of non-reciprocal interactions between flocking birds (Fig. 1a). To maintain a flocking arrangement, each bird adjusts its flight on the basis of the movement of the other birds in its immediate vicinity. Bird eyes, however, did not evolve to provide vision in all directions simultaneously. Instead, each bird responds only to other birds within its forward cone of vision[5]. If bird A is in the vision cone of bird B, then B responds to the motion of A; but if B is outside the vision cone of A, then A does not respond to the motion of B. In other words, the equivalence of action and reaction is lost.

In materials, non-reciprocal interactions are generally associated with the loss of a property called detailed balance, which arises when the reversal of a process occurs at the same rate as the process itself. When detailed balance is violated, the propagation of a signal in one direction differs from that in the opposite direction, either as a result of energy losses or because energy is injected by some type of pumping device. Imagine, for example, a system in which an elastic beam connects two devices that are sensitive to the compression or extension of the beam[6]. When the length of the beam is varied, the devices consume power and exert a torsional force on the beam. By contrast, a torque exerted on the beam by an outside force does not produce a response from the devices. Non-reciprocal interactions therefore arise from the asymmetry between torque and compression of the beam.

Non-reciprocal systems can undergo phase transitions in which a spontaneously broken symmetry is regained dynamically. For example, in the Vicsek model of flocking birds[3], a steady-state flock flies in a particular direction, thereby breaking spatial symmetry

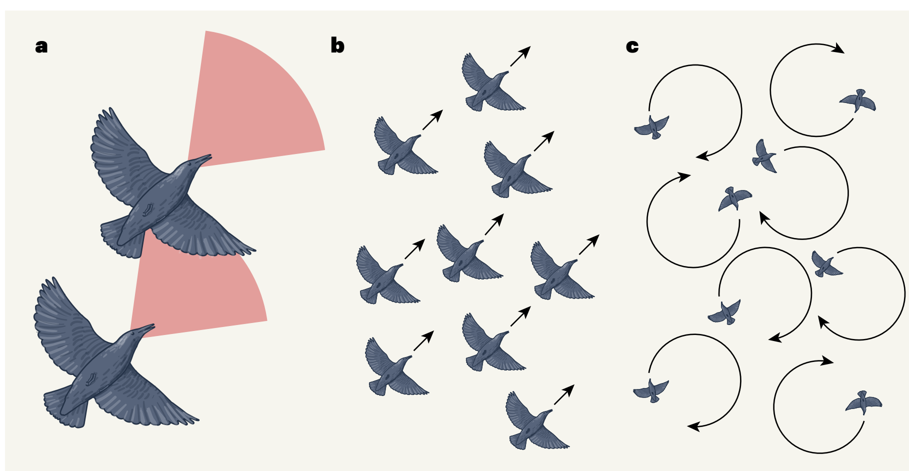

Figure 1 | Non-reciprocal interactions in bird flocking. a, Birds adjust their flight in response to other birds' movements to maintain flocking, but each individual responds only to the birds in its cone of vision (pink). Here, the lower bird responds to the bird above it, but not vice versa. Such interactions are said to be non-reciprocal. **b**, **c**, Non-reciprocal systems can undergo phase transitions in which a spontaneously broken symmetry is regained. For example, flocks spontaneously break continuous spatial symmetry if all the birds fly in one direction (**b**), but this symmetry can be restored if the birds enter a chiral state in which they all fly clockwise or anticlockwise in circles (**c**). Fruchart *et al.*[2] report a mathematical approach that describes how chiral states can emerge through the transition of the system through an exceptional point – a position in a numerical plot of a system's parameters at which several modes of the system coalesce into a single mode.

(Fig. 1b). When the interactions between birds are non-reciprocal, a state can emerge in which the birds fly in circles (Fig. 1c). The spatial symmetry in this state is restored because the birds fly in all directions. Importantly, this state has a chirality – the birds either all fly clockwise or all fly anticlockwise – that is stabilized by the many interactions between the birds. This stabilization prevents the system from flipping back and forth between the two chiralities, which would produce an average chirality of zero.

Fruchart *et al.* now show that the emergence of the chiral state occurs at a transition between symmetry and broken symmetry that is controlled by an exceptional point. By contrast, transitions in systems at equilibrium occur at mathematically distinct 'critical points' that are associated with the closing of an energy gap, which causes two distinct states of the system to have the same energy. The energy of a dynamic system can be described numerically by a mathematical function called a Hamiltonian, and fundamental modes of the system are characterized by vectors known as eigenvectors. The Hamiltonian of a system that has non-reciprocal interactions is non-Hermitian[1], which means that the eigenvectors are not fully independent. When the directions of these eigenvectors are varied by changing a control parameter of the system, two of the eigenvectors can coalesce at an exceptional point.

The authors show that in a many-body system, one of the two overlapping modes is known as a long-wavelength Goldstone mode, and is associated with the breaking of rotational invariance. In the case of a flock of birds, the Goldstone mode corresponds to a uniform movement of all birds along the flocking direction, whereas other modes control the relative motion of birds within the flock with respect to each other.

At the exceptional point, the complete overlap of the Goldstone mode with one of the other modes allows the system to freely switch between all possible ground states, instead of remaining trapped in one state. For the birds, this corresponds to the emergence of chiral rotation across the entire system. In other words, Fruchart *et al.* report how symmetry that was spontaneously broken on one side of the exceptional point can be dynamically restored.

Although exceptional points have received considerable attention in photonics[7], where they have been shown to describe properties such as the one-way transmission of light through a material, Fruchart and colleagues expand their use to many-body systems that are out of equilibrium. Indeed, the authors' findings apply to any system containing two key ingredients: non-reciprocal interactions and a spontaneously broken continuous symmetry. This opens up the possibility of engineering devices whose function depends on the behaviour of a non-reciprocal system that is close to its exceptional-point transition – by analogy to existing devices that exploit behaviour near ordinary phase transitions (such as a refrigerator, which repeatedly vaporizes and condenses its coolant).

For example, materials could be developed that exhibit one-way elasticity – that is, in which mechanical waves propagate undisturbed in one direction, but are totally reflected in the opposite direction. Devices could be engineered to produce coherent phonons, the mechanical equivalent of a laser beam. And it might be possible to develop mechanical strain cloaking, in which a portion of a material is fully isolated from vibrations or shocks.

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Stem cells

Relax to grow more hair

Rui Yi

A stress hormone has been found to signal through skin cells to repress the activation of hair-follicle stem cells in mice. When this signalling is blocked, hair growth is stimulated. Stressed humans, watch out. **See p.428**

When American football quarterback Aaron Rodgers told his fans to relax after his team's poor start one season, little did he know that he was also giving a hair-care tip. His advice is particularly helpful now, after a long pandemic year. About one-quarter of people who contract COVID-19 experience hair loss six months after the onset of symptoms[1], probably because of the systemic shock caused by the ordeal of infection and recovery. Chronic stress has long been associated with hair loss, but the underlying mechanism that links stress to the dysfunction of hair-follicle stem cells has been elusive. On page 428, Choi *et al.*[2] uncover the connection in mice.

Throughout a person's lifespan, hair growth cycles through three stages: growth (anagen), degeneration (catagen) and rest (telogen). During anagen, a hair follicle continuously pushes out a growing hair shaft. During catagen, hair growth stops and the lower portion of the hair follicle shrinks, but the hair (now known as a club hair) remains in place. During telogen, the club hair remains dormant for some time, eventually falling out. Under severe stress, many hair follicles enter telogen prematurely and the hair quickly falls out.

Hair-follicle stem cells (HFSCs) are located in a region of the hair follicle called the bulge. These cells have a crucial role in governing hair growth by interpreting both internal and external signals. For example, during telogen, HFSCs are kept in a quiescent state and so do not divide[3,4]. When hair growth is initiated in the next anagen phase, HFSCs are instructed to divide and produce progenitor cells. These progenitors then begin a journey of differentiation, generating several layers of hair follicles and, ultimately, the hair shaft.

Since HFSCs were identified in the bulge region more than 30 years ago[5–7], many regulatory molecules – such as gene-transcription factors and signalling proteins – have been shown to control the cells' quiescence and activation[3,4]. Nearly all of these regulators are produced by either HFSCs or their neighbouring cells, including dermal papilla cells, which usually function as a supportive 'niche' for HFSCs[8,9]. But how systemic conditions such as chronic stress affect the activity of HFSCs is incompletely understood.

To answer this question, Choi and colleagues first tested the role of adrenal glands – which produce stress hormones and constitute a key endocrine organ – in the regulation of hair growth, by surgically removing them from mice. Telogen phases were much shorter in the hair follicles of these animals (which the team dubbed ADX mice) than in control mice (less than 20 days compared with 60–100 days), and the follicles engaged in hair growth roughly three times as often. The authors were able to suppress this frequent hair growth and restore the normal hair cycle