# Jean Bourgain

## *(1954–2018)*

Prolific unifier of mathematics.

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Over the course of a wildly prolific career, Belgian mathematician Jean Bourgain would repeatedly enter some area, solve several of its outstanding problems and create an entirely new field of study in the process. His stomping ground was analysis — a discipline that uses estimates to capture the essence of messy mathematical quantities. But, unlike many mathematicians, he turned the powers of his discipline outward, using the lens of analysis to bring far-flung areas such as number theory and computer science into focus.

Bourgain, who died on 22 December, was born in 1954 in Ostend, Belgium. He earned a PhD in 1977 from the Vrije Universiteit Brussel, where he quickly started producing mathematics at the highest level. In 1985, he took up joint appointments at the University of Illinois at Urbana-Champaign and the Institute of Advanced Scientific Studies (IHES) near Paris.

Some of his notable early work concerned the geometry of high-dimensional convex shapes, such as how to find slices of the shape that are especially large or round. Many of these studies later found applications in high-dimensional data analysis — for example, to create feature-extraction algorithms.

In the early 1990s, Bourgain launched the modern era for dispersive, nonlinear partial differential equations. These model how waves spread out from each other over time, like ripples when a pebble drops into a lake. He developed a broad framework for understanding when such equations have well-behaved solutions, and how waves in this setting interact (as when a child's kicks amplify or dampen the motion of a swing).

In 1994, these advances won Bourgain a Fields Medal, widely seen as the highest honour awarded in mathematics. But Bourgain — who, in that same year, began an appointment at the Institute for Advanced Study (IAS) in Princeton, New Jersey, where he spent the rest of his career — was just getting started.

Mathematics poured out of him at a dizzying rate: he wrote more than 500 papers over his lifetime, easily 5–10 times what a typical research mathematician might produce.

Telling Bourgain about one's research could be an intimidating prospect. This was not because of his personality, which colleagues describe as kind, optimistic and intellectually generous. Rather, it was because Bourgain would often pull some novel technique out of his hat to solve a problem the other mathematician had been struggling with for months. (One mathematician ripped up a manuscript he had planned to submit for publication, after Bourgain — off the cuff — came up with an easy proof of the paper's main result.)

Bourgain lived and breathed mathematics. A night owl who travelled frequently back and forth between the IAS and California, where his wife and son lived, he would often, when at Princeton, show up at the IAS shortly before afternoon tea and work with collaborators until well past midnight. Then, after the others had gone to bed, he would continue into the night. His collaborators would often wake up to an e-mail from Bourgain sent at 4 a.m., containing the solution to a problem that, the night before, had seemed as if it would take weeks to crack.

In the papers he co-authored, mathematicians say, it is easy to spot which sections Bourgain wrote. As with his solo papers, they are impenetrable thickets, making giant conceptual leaps with little by way of explanation. One page of a Bourgain paper might take weeks for the uninitiated to decipher. But the deep ideas and techniques within were ample reward for the persevering reader.

Asking Bourgain to make his papers more readable would have been a disservice to mathematics, says Alex Kontorovich, a frequent collaborator of Bourgain's at Rutgers University in Piscataway, New Jersey. "He needed to be totally free," Kontorovich explains. "He was going to connect so many fields — so let him go do it, and people after him will come pick up the pieces."

In the early 2000s, Bourgain settled a question about addition and multiplication that is simple to state, but hard to prove. From a given set of numbers, you can build two new sets by listing the sums and the products of all the pairs. Each of these two lists generally contains some redundancies; for example, 8 equals both $3 + 5$ and $1 + 7$. But Bourgain and his collaborators showed, for a variety of different number systems, that the sums list and the products list usually cannot both have a high degree of redundancy — at least one will have significantly more numbers than the original set. This elementary statement about numbers has been used in computer science to show that a wide class of networks are 'expanders', valuable tools for generating random structures, such as a well-shuffled deck of cards.

Bourgain was diagnosed with pancreatic cancer in late 2014. Between blasts of chemotherapy, the pace of his research scarcely slackened. In late 2015, for instance, Bourgain and his collaborators settled an 80-year-old conjecture in number theory. It concerned an inequality that governs a host of questions about the properties of whole numbers, such as which can be expressed as a sum of four perfect squares or nine perfect cubes.

Many mathematicians had expected the solution to come from number theory. Characteristically, Bourgain tackled the problem by means of harmonic analysis. Lists of numbers such as perfect powers can be converted into waves of those frequencies. Starting there, Bourgain and his collaborators proved a much more general result about such waves than was needed to prove the conjecture.

The flow of ideas has ended, but the flow of papers probably has not. Mathematician Peter Sarnak, Bourgain's colleague at the IAS, predicts that as many as 50 more might be published posthumously by Bourgain's collaborators. Together with his densely written earlier papers, these will give mathematicians treasures to mine for decades to come. ■

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